MATH601 Spring 2008
Exam 6 Solutions

1. Calculate $1/2 + 3/4$, using the definition of $+$.

   Recall that $1 = \prec 0 | \succ$, $1/2 = \prec 0 | 1 \succ$, $3/4 = \prec 1/2 | 1 \succ$, $3/2 = \prec 1 | 2 \succ$.
   $1/2 + 3/4 = \prec 0 + 3/4, 1/2 + 1/2 | 1 + 3/4, 1 + 1/2 \succ$
   $1/2 + 1/2 = \prec 0 + 1/2, 0 + 1/2 | 1 + 1/2, 1 + 1/2 \succ \quad 1 + 1/2 = \prec 0 + 1/2, 0 + 1 | 1 + 1 \succ$
   $1 + 1 = \prec 0 + 1, 0 + 1 | \succ = \prec 1 | \succ = 2$
   Hence $1 + 1/2 = \prec 1/2, 1 | 2 \succ = 3/2$, by the exercise where $\prec a, b | c \succ = \prec b | c \succ$ if $a < b$.
   Hence $1/2 + 1/2 = \prec 1/2 | 3/2 \succ = 1$ by the Seniority Principle.
   $1 + 3/4 = \prec 0 + 3/4, 1 + 1/2 | 1 + 1 \succ = \prec 3/4, 3/2 | 2 \succ = 7/4$ by the Seniority Principle.
   Finally, $1/2 + 3/4 = \prec 3/4, 1 | 7/4, 3/2 \succ = 5/4$ by the Seniority Principle.

2. For all surreal numbers y, prove that $y \not> y$.

   Method 1: $y \not> y$ if and only if $y \geq y$, which was previously proved (Exercise 7 from Handout 13, then again in Thm 1 from Handout 14).

   Method 2: Proof by surreal induction. Suppose that $y > y$. Case (i): There is $a \in R(y)$ with $y \geq a$. Hence we have $a > y \geq a$, and by Thm 3 from Handout 14 (transitivity), we have $a > a$, which is impossible by surreal induction. Case (ii): There is $b \in L(y)$ with $b \geq y$. Hence we have $b \geq y > b$, and by Thm 3 from Handout 14, $b > b$, which is again impossible by surreal induction.

   Method 3: Suppose that $y > y$. Then, by definition of $>$, either (i) there is some $a \in R(y)$ with $y \geq a$; or (ii) there is some $b \in L(y)$ with $b \geq y$. But both of these violate the Seniority Principle: $y^L < y < y^R$. Hence $y \not> y$.

3. Exam grades: 100, 98, 97, 90, 89, 80, 76, 76, 72, 53