MATH 579 Exam 4 Part II; 3/9/10

Please read the exam instructions.

No books or notes are permitted for this exam; calculators are permitted though. Please write your answers on separate paper, indicate what work goes with which problem, and put your name or initials on every sheet. Cross out work you do not wish graded; incorrect work can lower your grade, even compared with no work at all. Keep this sheet for your records. Show all necessary work in your solutions; if you are unsure, show it. Simplify all numerical answers to be integers, if possible. Please attach part I to your solutions. You have 35 minutes. If you wish, you may hand in solutions to all six problems (part I and II) on the next class day, March 11. For more details, see the syllabus.

PART II: Choose three problems only from these five.

1. (5-8 points) Prove that for $n \in \mathbb{N}_0$, $3^n = \sum_{k=0}^{n} 2^k \binom{n}{k}$.
2. (5-10 points) Prove that for $n \in \mathbb{N}$, $\binom{2n}{n} < 4^n$.
3. (5-10 points) Recall that a northeastern lattice path consists of steps $(0,1)$ and $(1,0)$ in some order. How many northeastern lattice paths are there from $(0,0)$ to $(20,10)$ that do not pass through $(15,5)$?
4. (5-10 points) Prove that for $k, m, n \in \mathbb{N}$, $\binom{n+m}{k} = \sum_{i=0}^{k} \binom{n}{i}\binom{m}{k-i}$.
5. (5-12 points) When we expand $(x_1+x_2+\cdots+x_m)^n$ fully, what is the largest coefficient?