MATH 579 Exam 9: 5/12/9

Please read the exam instructions.

Please write your answers on separate paper, indicate clearly what work goes with which problem, and put your name or initials on every sheet. Cross out work you do not wish graded; incorrect work can lower your grade, even compared with no work at all. Keep this list of problems for your records. Show all necessary work in your solutions; if you are unsure, show it. Simplify all numerical answers to be integers, if possible. You may earn extra credit by submitting BY NOON ON THURSDAY (May 14), revised solutions to all six problems – for more details, please see the syllabus. Your graded exams may be picked up after noon Thursday. This exam is out of 40 points maximum.

PART I: Choose three problems only from the first five.

1. (5-8 points) Let a_n denote the number of ways to pay n cents using pennies, nickels, dimes and/or quarters. For example, $a_5 = 2$ (either five pennies or one nickel). Find the generating function for a_n.
2. (5-10 points) Let a_n be the number of subsets of $[n]$ in which the difference between any two elements is at least four. Find the generating function for a_n.
3. (5-10 points) Use generating functions to solve the recurrence $a_0 = 1, a_n = 3a_{n-1}+2^{n-1}$.
4. (5-10 points) A permutation is called *indecomposable* if its one-line notation cannot be split into two parts such that every number in the first part is smaller than every number in the second part. Let a_n denote the number of indecomposable permutations of length n, with $a_0 = 0$. Find the generating function for a_n. You may use the generating function $B(x) = \sum_{n\geq 0} n!x^n$, the generating function of all permutations.
5. (5-12 points) Find the generating function for $a_n = n^3$.

PART II: Choose either problem 6 or problem 7.

6. (5-10 points) Let a_n denote the number of ways to place n identical nonattacking rooks on an $n \times n$ board, with no rooks on the diagonal. Prove that this equals $D(n)$, the number of derangements of $[n]$. Use this to calculate a_6.
7. Do both problems that you skipped from Part I. Your score will be the lower of the two. Be sure to indicate which two problems you are counting as problem 7.