MATH 579 Exam 8: 4/30/9

Please read the exam instructions.

Please write your answers on separate paper, indicate clearly what work goes with which problem, and put your name or initials on every sheet. Cross out work you do not wish graded; incorrect work can lower your grade, even compared with no work at all. Keep this list of problems for your records. Show all necessary work in your solutions; if you are unsure, show it. Simplify all numerical answers to be integers, if possible. You may earn extra credit by submitting by the next class period (May 5), revised solutions to all six problems – for more details, please see the syllabus. This exam is out of 40 points maximum.

PART I: Choose three problems only from the first five.

1. (5-8 points) Let a_n denote the number of ways to color the squares of a $1 \times n$ chessboard using the colors red, white, and blue, so that no two white squares are adjacent.
2. (5-10 points) Solve the recurrence $a_0 = a_1 = 0, a_n = a_{n-1} + 2a_{n-2} + n \quad (n \geq 2)$.
3. (5-10 points) Codewords from the alphabet $\{0, 1, 2, 3\}$ are called *legitimate* if they have an even number of 0's. How many legitimate codewords are there, of length k?
4. (5-10 points) Prove that $a_n = \alpha 2^n + \beta$ is the general solution to $a_n = 3a_{n-1} - 2a_{n-2}$. In other words, prove that for all possible initial conditions (a_0, a_1), there is exactly one (α, β) that gives the recurrence (relation+initial conditions).
5. (5-12 points) Solve the recurrence $a_1 = 2, na_n + na_{n-1} - a_{n-1} = 2^n \quad (n \geq 2)$.
   HINT: Set $b_n = na_n$.

PART II: Choose either problem 6 or problem 7.

6. (5-10 points) How many ways are there to place five (identical) nonattacking rooks on a 5×5 chessboard, with no rooks occupying places $(1, 1), (2, 2), (3, 3), (4, 4)$?
   Note: the forbidden squares are four of the five diagonal squares.
7. Do both problems that you skipped from Part I. Your score will be the lower of the two. Be sure to indicate which two problems you are counting as problem 7.