

**MATH 579 Exam 7: 4/21/9**  
Please read the exam instructions.

Please write your answers on separate paper, indicate clearly what work goes with which problem, and put your name or initials on every sheet. Cross out work you do not wish graded; incorrect work can lower your grade, even compared with no work at all. Keep this list of problems for your records. Show all necessary work in your solutions; if you are unsure, show it. Simplify all numerical answers to be integers, if possible. You may earn extra credit by submitting by the next class period (Apr. 23), revised solutions to all six problems – for more details, please see the syllabus. This exam is out of 40 points maximum.

**PART I: Choose three problems only from the first five.**

1. (5-8 points) How many integers in  $[1, 900000]$  are relatively prime to 900000?
2. (5-10 points) How many four-digit integers are not divisible by 6, 7, or 8?
3. (5-10 points) How many  $n$ -permutations are there with exactly one cycle of length one?
4. (5-10 points) How many 10-permutations are there with exactly one descent?
5. (5-12 points) How many solutions are there to  $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 8$  where  $0 \leq x_i \leq i$ ?

**PART II: Choose either problem 6 or problem 7.**

6. (5-10 points) How many ways are there to place five (identical) nonattacking rooks on a  $5 \times 5$  chessboard, with no rooks on the diagonal?
7. Do both problems that you skipped from Part I. Your score will be the lower of the two. Be sure to indicate which two problems you are counting as problem 7.