MATH 579 Exam 5: 3/24/9

Please read the exam instructions.

Please write your answers on separate paper, indicate clearly what work goes with which problem, and put your name or initials on every sheet. Cross out work you do not wish graded; incorrect work can lower your grade, even compared with no work at all. Keep this list of problems for your records. Show all necessary work in your solutions; if you are unsure, show it. Simplify all numerical answers to be integers, if possible. You may earn extra credit by submitting by the next class period (Mar. 26), revised solutions to all six problems – for more details, please see the syllabus. This exam is out of 40 points maximum.

PART I: Choose three problems only from the first five.

1. (5-8 points) Calculate the number of compositions of 14 into an even number of even parts.
2. (5-10 points) For all $n \in \mathbb{N}$, determine $S(n, n-2)$.
3. (5-10 points) Calculate $S(8, 3)$.
4. (5-10 points) Let a_n denote the number of compositions of n where each part is larger than 1. Find a formula relating a_n, a_{n-1}, a_{n-2}.
5. (5-12 points) For all $l, m, n \in \mathbb{N}_0$, prove that $\sum_k \binom{n}{k} S(k,l) S(n-k,m) = S(n, l+m)\binom{l+m}{l}$.

PART II: Choose either problem 6 or problem 7.

6. (5-10 points) For every prime p, prove that $B(p) \equiv 2 \pmod{p}$. Equivalently, prove that p divides $B(p) - 2$.
7. Do both problems that you skipped from Part I. Your score will be the lower of the two. Be sure to indicate which two problems you are counting as problem 7.