MATH 579 Exam 4: 3/12/9
Please read the exam instructions.

Please write your answers on separate paper, indicate clearly what work goes with which problem, and put your name or initials on every sheet. Cross out work you do not wish graded; incorrect work can lower your grade, even compared with no work at all. Keep this list of problems for your records. Show all necessary work in your solutions; if you are unsure, show it. Simplify all numerical answers to be integers, if possible. You may earn extra credit by submitting by the next class period (Mar. 17), revised solutions to all six problems – for more details, please see the syllabus.

PART I: Choose three problems only from the first five.

1. (5-8 points) For all $n \in \mathbb{N}$, prove that $\sum_k \binom{n}{k}\binom{2n}{n-k} = \binom{3n}{n}$.

2. (5-10 points) A northeastern lattice path is a path consisting of $(1,0)$ and $(0,1)$ steps. How many such paths are there from $(0,0)$ to $(10,10)$ that do not pass through $(1,1)$?

3. (5-10 points) Which monomial term(s) of $(x+y+z)^{16}$ has the largest coefficient? What is that coefficient?

4. (5-10 points) For all $n \in \mathbb{N}$, calculate $\sum_{k \text{ odd}} \binom{n}{k}3^k$.

5. (5-12 points) For all $k \in \mathbb{Z}$, prove that $\binom{n-1}{k-1}\binom{n}{k+1}\binom{n+1}{k} = \binom{n-1}{k}\binom{n+1}{k+1}\binom{n}{k-1}$.

PART II:

6. (10-20 points) For $m, n \in \mathbb{N}$, prove that $\frac{1}{m!} = \lim_{n\to\infty} \binom{m+n}{n}n^{-m}$.