Math 579 Exam 7 (part I): 4/12/7

Please read the exam instructions.

Please write your answers on **separate paper**, indicate clearly what work goes with which problem, and put your name on every sheet. Cross out work you do not wish graded; incorrect work can lower your grade, even compared with no work at all. Keep this list of problems for your records. Show all necessary work in your solutions; if you are unsure, show it. Each problem is worth a minimum of 5 points, and a maximum that is indicated. You have 40 minutes. *Choose three problems. Simplify all numerical answers.*

1. (8 points) How many three-letter words do not have consecutive identical letters?
   Note: Ignore the English language; 'gxn' is a valid word.

2. (10 points) How many positive integers less than or equal to $1001 = 7 \times 11 \times 13$ are relatively prime to 1001?

3. (10 points) How many three-digit positive integers are divisible by at least one of six and eight?

4. (10 points) Show an example of three subsets A, B, C of the natural numbers $\mathbb{N}$ so that:

   (a) $|A \cap B| = |A \cap C| = |B \cap C| = \infty$

   (b) $|A \cap B \cap C| = 0$

   (c) $A \cup B \cup C = \mathbb{N}$

5. (12 points) Four married couples (8 people) get in one line at a buffet. To be sociable, they decide that no two married people will stand next to each other in line. How many ways can this be done?