

**Math 579 Exam 1 (part I): 1/30/7**

Please read the exam instructions.

Please write your answers on **separate paper**, indicate clearly what work goes with which problem, and put your name on every sheet. Cross out work you do not wish graded; incorrect work can lower your grade, even compared with no work at all. Keep this list of problems for your records. Show all necessary work in your solutions; if you are unsure, show it. Each problem is worth a minimum of 5 points, and a maximum that is indicated. You have 40 minutes. *Choose three problems.*

1. (8 points) Find all triples of distinct positive integers  $a, b, c$  such that  $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 1$ .
2. (10 points) Prove that among 51 integers, there are always two so that either their sum or their difference is a multiple of 99.
3. (10 points) Prove that among 1001 distinct integers chosen from  $[1, 2000]$ , there are always two that are relatively prime (have greatest common divisor 1).
4. (10 points) Prove that the sequence  $2006, 20062006, 200620062006, \dots$  has an element that is a multiple of 2007.
5. (12 points) A thousand pennies are initially divided into four piles. They are then rearranged into six piles. Prove that at least three pennies end up in a smaller pile, and give an example in which exactly three pennies do so.