Math 524 Exam 5: 10/16/8
Please read the exam instructions.

Notes, books, papers, calculators and electronic aids are all forbidden for this exam. Please write your answers on **separate paper**, indicate clearly what work goes with which problem, and put your name on every sheet. Cross out work you do not wish graded; incorrect work can lower your grade, even compared with no work at all. Keep this list of problems for your records. Show all necessary work in your solutions; if you are unsure, show it. Each problem is worth 10 points. You have approximately 30 minutes.

1. Suppose that A, B are square, diagonalizable matrices satisfying $AB = BA + I$. Without using Thm. 4.10, prove that they are not simultaneously diagonalizable. (Note: Thm 4.10 says that A, B commute if and only if they are simultaneously diagonalizable).

   The remaining problems all concern the matrix $A = \begin{pmatrix} 1 & -1 & -1 \\ -1 & 1 & -1 \\ 2 & 2 & 4 \end{pmatrix}$.

2. Find all eigenvalues of A; give a basis for each eigenspace.
   HINT: each column sums to 2.

3. Find a basis for $\mathbb{R}^3$ consisting of "power vectors" (generalized eigenvectors) of A.

4. Write A in Jordan canonical form. You need not find the corresponding change-of-basis matrix.

5. Evaluate $e^{At}\bar{u}$, for each $\bar{u}$ in the basis you gave in question 3. above.