Math 524 Exam 4 Makeup: 10/9/8

You will keep the higher score of this and the original.

Please read the exam instructions.

Notes, books, papers, calculators and electronic aids are all forbidden for this exam. Please write your answers on **separate paper**, indicate clearly what work goes with which problem, and put your name on every sheet. Cross out work you do not wish graded; incorrect work can lower your grade, even compared with no work at all. Keep this list of problems for your records. Show all necessary work in your solutions; if you are unsure, show it. Each problem is worth 10 points. You have approximately 30 minutes.

For each of the following vector spaces V and linear operators L:

1. Find all eigenvalues.
2. Find a basis for each eigenspace.
3. Determine all algebraic and geometric multiplicities.
4. Is the operator diagonalizable?

A. $V = \mathbb{R}^3$, $L(x) = \left[\begin{smallmatrix} 0 & 8 & -4 \\ 0 & 2 & 0 \\ 1 & -4 & 4 \end{smallmatrix}\right] x$

B. $V = M_{2,2}(\mathbb{R})$, the set of all 2×2 real matrices. We have $V = W_1 + W_2$, an internal direct sum, where $W_1 = \{A : (A)_{12} = 0\}$ is the subspace of lower triangular matrices, and $W_2 = \{A : A = -A^T\}$ is the subspace of skew-symmetric matrices. L is the operator that projects from V to W_2.

C. $V = C^1(\mathbb{R})$, the set of continuously differentiable functions on the real line, $L(f) = (t+1)\frac{df}{dt}$