Fall 2009 Math 522 Final Exam

Please read the exam instructions.

Please write your answers on **separate paper**, indicate clearly what work goes with which problem, and put your name on every sheet. Notes, calculators, and the textbook are all permitted. Cross out work you do not wish graded; incorrect work can lower your grade, even compared with no work at all. Keep this list of problems for your records. Show all necessary work in your solutions; if you are unsure, show it. You will earn between 7 and 14 points on each problem (and a 2 point bonus because I like you). You have 120 minutes.

Choose 7 of the following 8 problems to complete. You may do all 8 for a bonus.

1. Calculate and simplify $\binom{15/7}{5}$.
2. Solve the congruence $67x \equiv 387 \pmod{440}$ using the Chinese Remainder Theorem.
3. Solve the congruence $67x \equiv 387 \pmod{440}$ directly (without CRT), by first finding $67^{-1} \pmod{440}$.
4. Determine whether or not 105 is a quadratic residue modulo 151. Be sure to indicate at all times whether you are using Legendre or Jacobi symbols.
5. Prove that every natural number is congruent, modulo 9, to the sum of its digits.
6. Prove that, for all natural n, $1\cdot 2+2\cdot 3+3\cdot 4+\cdots+n(n+1) = \frac{n(n+1)(n+2)}{3}$.
7. We define $\xi(n) = \prod_{p|n} p^2$. Prove that ξ is multiplicative, and characterize those n for which $\xi(n) = n$.
8. Prove that there is no primitive root modulo $15251 = 101 \cdot 151$. [You may not use the theorem from class that classifies which integers possess primitive roots.]