

Math 522 Exam 2 Solutions

1. Find all pairs of positive integers (x, y) such that $21x + 15y = 363$, if any exist.

We find $3 = \gcd(21, 15)$, and $3|363$, so integer solutions exist (though not necessarily positive integer solutions). We turn to our old favorite, trial and error. $x = 1$ gives $y = 114/5$, $x = 2$ gives $y = 107/5$, but $x = 3$ gives the solution $(3, 20)$. To find other solutions, add $5 = 15/3$ to x and subtract $7 = 21/3$ from y , to get the other two solutions $(8, 13)$ and $(13, 6)$. No other positive integer solutions exist, since all other integer solutions have either x or y negative.

2. Let m, n, a, b be integers, with $b \neq 0$. Suppose that $\gcd(m, ab) = 1$. Prove that $\gcd(ma + nb, b) = \gcd(a, b)$.

Note that $\gcd(m, a) \in CD(m, ab)$, so $\gcd(m, a) | \gcd(m, ab) = 1$. Similarly, $\gcd(m, b) = 1$. By part 5 of the GCD theorem proved in class, since $\gcd(m, a) = 1$, we must have $\gcd(m, b) \gcd(a, b) = \gcd(am, b)$. However, since $\gcd(m, b) = 1$, we must in fact have $\gcd(a, b) = \gcd(am, b)$. By part 3 of the same theorem, we have $\gcd(am, b) = \gcd(am + nb, b)$. Combining these two gives $\gcd(a, b) = \gcd(am + nb, b)$, as desired.

Alternate solution: Let $d = \gcd(ma, b)$, $e = \gcd(a, b)$. First, since $e \in CD(ma, b)$ we have $e \leq d$. Second, we multiply the two equations $\alpha m + \beta ab = 1$, $\gamma a + \delta b = e$, to get $am(\alpha\gamma) + b(\beta\gamma a^2 + \beta\delta ab + \alpha\delta m) = e$. Hence $e \in PS(ma, b)$ so $e \geq d$. Hence $e = d$.

3. Exam grades: 98, 90, 90, 88, 85, 83, 81, 80, 80, 77, 75, 75, 75, 73, 70