

**Math 254-1 Exam 9: 11/17/8**

Please read the exam instructions.

Notes, books, papers, calculators and electronic aids are all forbidden for this exam. Please write your answers on **the attached page only** (front and back if necessary). Indicate clearly what work goes with which problem. Cross out work you do not wish graded; incorrect work can lower your grade. You may use this first page as scratch paper; keep it for your records. Show all necessary work in your solutions; if you are unsure, show it. Extra credit may be earned by handing in revised work in class on Wednesday 11/19; for details see the syllabus. Each problem is worth 10 points; your total will be doubled to fit the standard 100 point scale. You have approximately 30 minutes.

1. Carefully define the term “spanning”. Give two examples in  $\mathbb{R}^2$ .
2. Consider the basis  $S = \{(1, -2), (2, -5)\}$  of  $\mathbb{R}^2$ , and the linear operator  $F(x, y) = (2x + 3y, 4x - 5y)$ . Find the matrix representation  $[F]_S$ .
3. Prove that, for any square matrices  $A, B$ , if  $A$  is similar to  $B$ , then  $B$  must be similar to  $A$ .

For the last two questions, set  $V$  to be the vector space of functions that have as a basis  $S = \{1, \sin \theta, \cos \theta, \sin 3\theta, \cos 3\theta\}$ .

4. Let  $D$  be the differential operator on  $V$ ,  $D(f(\theta)) = f'(\theta)$ . Find the matrix representation  $[D]_S$ .
5. Let  $L$  be the operator on  $V$  given by  $L(f(\theta)) = f''(\theta) - f(\theta)$ . Find the matrix representation  $[L]_S$ .

**Please hand in ONLY the second page; keep this first page.**

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Please write all solutions on this page (front and back if necessary).