## Math 254-2 Exam 4: 10/14/8
Please read the exam instructions.

Notes, books, papers, calculators and electronic aids are all forbidden for this exam. Please write your answers on **the attached page only** (front and back if necessary). Indicate clearly what work goes with which problem. Cross out work you do not wish graded; incorrect work can lower your grade. You may use this first page as scratch paper; keep it for your records. Show all necessary work in your solutions; if you are unsure, show it. Extra credit may be earned by handing in revised work in class on Thursday 10/16; for details see the syllabus. Each problem is worth 10 points; your total will be scaled to the standard 100 point scale. You have approximately 30 minutes.

1. Carefully state the definition of "subspace". Give two examples from $\mathbb{R}^2$.

2. Carefully state five of the eight vector space axioms.

3. Let $S = \{f(x) : f(17) = 0\} \subseteq \mathbb{R}[x]$ be the set of all polynomials that are zero at $x = 17$. Prove that this is a vector space.

4. Determine, with justification, whether $(1, 1, 1)$ is in $Span(S)$, for $S = \{(1, 2, 1), (0, 3, 2), (2, 1, 0)\}$.

5. Let $W_1 = Span(S)$, for $S = \{\left(\begin{smallmatrix} 1 & 1 \\ 0 & 0 \end{smallmatrix}\right), \left(\begin{smallmatrix} 0 & 0 \\ 1 & 1 \end{smallmatrix}\right)\}$. Let $W_2 = Span(T)$, for $T = \{\left(\begin{smallmatrix} 1 & 0 \\ 0 & 0 \end{smallmatrix}\right), \left(\begin{smallmatrix} 0 & 0 \\ 1 & 0 \end{smallmatrix}\right)\}$. Prove that $W_1 \oplus W_2 = M_{22}(\mathbb{R})$ (the set of all $2 \times 2$ matrices).

**Please hand in ONLY the second page; keep this first page.**

Please write all solutions on this page (front and back if necessary).