

Math 254-1 Exam 4: 10/13/8

Please read the exam instructions.

Notes, books, papers, calculators and electronic aids are all forbidden for this exam. Please write your answers on **the attached page only** (front and back if necessary). Indicate clearly what work goes with which problem. Cross out work you do not wish graded; incorrect work can lower your grade. You may use this first page as scratch paper; keep it for your records. Show all necessary work in your solutions; if you are unsure, show it. Extra credit may be earned by handing in revised work in class on Wednesday 10/15; for details see the syllabus. Each problem is worth 10 points; your total will be scaled to the standard 100 point scale. You have approximately 30 minutes.

1. Carefully state the definition of “subspace”. Give two examples from \mathbb{R}^2 .
2. Carefully state five of the eight vector space axioms.
3. Let $S = \{f(x) : f(3) = 0\} \subseteq \mathbb{R}[x]$ be the set of all polynomials that are zero at $x = 3$. Prove that this is a vector space.
4. Determine, with justification, whether $(1, 1, 1)$ is in $\text{Span}(S)$, for $S = \{(1, 2, 3), (2, 0, 1), (-3, 2, 1)\}$.
5. Let $W_1 = \text{Span}(S)$, for $S = \left\{\begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix}\right\}$. Let $W_2 = \text{Span}(T)$, for $T = \left\{\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}\right\}$. Prove that $W_1 \oplus W_2 = M_{22}(\mathbb{R})$ (the set of all 2×2 matrices).

Please hand in ONLY the second page; keep this first page.

ID Code: _____

Please write all solutions on this page (front and back if necessary).