

**Math 254-1 Exam 4: 10/13/8**

Please read the exam instructions.

Notes, books, papers, calculators and electronic aids are all forbidden for this exam. Please write your answers on **the attached page only** (front and back if necessary). Indicate clearly what work goes with which problem. Cross out work you do not wish graded; incorrect work can lower your grade. You may use this first page as scratch paper; keep it for your records. Show all necessary work in your solutions; if you are unsure, show it. Extra credit may be earned by handing in revised work in class on Wednesday 10/15; for details see the syllabus. Each problem is worth 10 points; your total will be scaled to the standard 100 point scale. You have approximately 30 minutes.

1. Carefully state the definition of “subspace”. Give two examples from  $\mathbb{R}^2$ .
2. Carefully state five of the eight vector space axioms.
3. Let  $S = \{f(x) : f(3) = 0\} \subseteq \mathbb{R}[x]$  be the set of all polynomials that are zero at  $x = 3$ . Prove that this is a vector space.
4. Determine, with justification, whether  $(1, 1, 1)$  is in  $\text{Span}(S)$ , for  $S = \{(1, 2, 3), (2, 0, 1), (-3, 2, 1)\}$ .
5. Let  $W_1 = \text{Span}(S)$ , for  $S = \left\{\begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix}\right\}$ . Let  $W_2 = \text{Span}(T)$ , for  $T = \left\{\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}\right\}$ . Prove that  $W_1 \oplus W_2 = M_{22}(\mathbb{R})$  (the set of all  $2 \times 2$  matrices).

**Please hand in ONLY the second page; keep this first page.**

ID Code: \_\_\_\_\_

Please write all solutions on this page (front and back if necessary).