

Math 254 Exam 9: 11/28/6

Please read the exam instructions.

Notes, books, papers, calculators and electronic aids are all forbidden for this exam. Please write your answers on **separate paper**, indicate clearly what work goes with which problem, and put your name on every sheet. Cross out work you do not wish graded; incorrect work can lower your grade, even compared with no work at all. Keep this list of problems for your records. Show all necessary work in your solutions; if you are unsure, show it. Extra credit may be earned by handing in revised work in class on Thursday 11/30; for details see the syllabus. Each problem is worth 10 points.

1. Carefully define the term “basis”. Give two examples in \mathbb{R}^2 .
2. Consider the basis $S = \{(1, -2), (2, -5)\}$ of \mathbb{R}^2 , and the linear operator $F(x, y) = (2x + 3y, 4x - 5y)$. Find the matrix representation $[F]_S$.
3. Let V be the vector space of functions that have as a basis $S = \{1, \sin \theta, \cos \theta, \sin 2\theta, \cos 2\theta\}$. Let D be the differential operator on V . Find the matrix representation $[D]_S$.
BONUS: What is the nullity and rank of D ?
4. Set $A = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}$. Find two other matrices similar to A .
5. Prove that, for any square matrices A, B , if A is similar to B , then B must be similar to A .