Fall 2007 Math 151 (sections 5,6,7,8) Midterm

Please read the following directions:

Please write legibly, with plenty of white space. Please put your answers in the designated areas. To get credit, you must also show adequate work to justify your answers. If unsure, show the work. Each problem has multiple parts; to get full credit you must answer each part. Give decimal approximations to four places after the decimal point. All problems are worth 7-14 points. You may use calculators, but no books or notes. This exam will last 75 minutes; pace yourself accordingly. If you are done early, you may leave – but NOT during the last five minutes of the exam, during which you are asked to remain quiet and in your seat. Good luck!

Problem	Min Score	Your Score	Max Score
1. Area	7		14
2. Centroid	7		14
3. Arc Length	7		14
4. Necklace	7		14
5. Volume	7		14
6. Separable DE	7		14
7. Linear DE	7		14
Freebie	2	2	2
Total:	51		100

This page is extra scratch paper, or just in case you mess up a problem and need a fresh page. If this is a rewrite, clearly indicate which problem this is meant to be.

Problem 1. We seek the area between the curves $y = x^2$ and $y = 13 - (x - 1)^2$. Set up an integral that gives this area, and evaluate it.

Integral:

Area (exact answer):

Area (decimal approx.):

Problem 2. We seek the centroid (center of mass) of the region bounded above by $y = 1/x^5$, below by the x-axis, and on the left by x = 1. Set up integrals that give the centroid, and evaluate them.

\bar{x} (exact	answer)	1:
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 \bar{x} (decimal approx.):

 \bar{y} (exact answer):

 \bar{y} (decimal approx.):

Problem 3. We seek the length of the curve $y = 3^x$ between (0, 1) and (1, 3). Set up an integral that gives this length, and estimate it using Simpson's rule with n = 2.

Integral:

Decimal Approx. (using Simpson's rule):

Problem 4. We rotate the curve $y = \sin x$ about the x axis. The result looks like a necklace of (infinitely many) identical beads. We seek the volume of one bead. Set up an integral that gives this volume, and evaluate it.

Integral:

Volume (exact answer):

Volume (decimal approx.):

Problem 5. Consider the region bounded by $x = 0, x = 1, y = 0, y = \frac{1}{6-x-x^2}$. We rotate this region about the y axis, and seek the volume of the resulting shape. Set up an integral that gives this volume, and evaluate it.

Integral:

Volume (exact answer):

Volume (decimal approx.):

Problem 6. Consider the differential equation $\frac{\delta}{dy/dx} = xye^x$, with initial condition y(0) = 1. Find the general solution, the specific solution (using the initial condition), and a decimal approximation to y(2).

General Solution:

Specific Solution:

Decimal Approx. to y(2):

9

Problem 7. Consider the differential equation $\frac{dy}{dx} + \frac{2y}{x} = \frac{(\sin 3x)}{x^2}$, with initial condition $y(\pi) = 0$. Find the general solution, the specific solution (using the initial condition), and a decimal approximation to $y(2\pi)$.

General Solution:

Specific Solution:

Decimal Approx. to $y(2\pi)$: