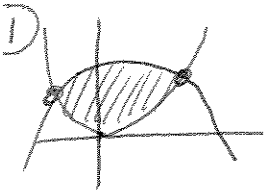
Math 151 Midterm Solutions Fall 2007

(1) 

$x^2 = 13-(x-1)^2$

$x^2 - x - 6 = 0$

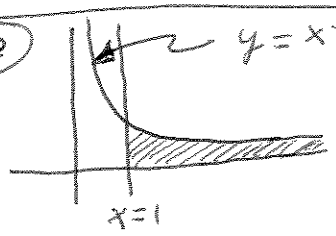
$x = -2, 3$

$$\int_{-2}^{3} 13-(x-1)^2 - x^2\,dx$$

$$= \int_{-2}^{3} 12+2x-x^2\,dx$$

$$= 12x + x^2 - \tfrac{2}{3}x^3 \Big]_{-2}^{3}$$

$$= 125/3 \approx 41.6667$$

(2) 

Region is infinitely wide.

$f(x) = x^{-5}$

$$A = \int_1^\infty f(x)\,dx = \lim_{M\to+\infty} -\tfrac{1}{4}x^{-4}\Big]_1^M = 1/4$$

$$\bar{x} = \frac{1}{A}\int_1^\infty x f(x)\,dx = \lim_{M\to+\infty} \frac{1}{A}\left(-\tfrac{1}{3}x^{-3}\right)\Big]_1^M = \frac{1/3}{1/4} = 4/3 \approx 1.3333$$

$$\bar{y} = \frac{1}{2A}\int_1^\infty f(x)^2\,dx =$$

$$= \lim_{M\to+\infty} \frac{1}{2A}\left(-\tfrac{1}{9}x^{-9}\right)\Big]_1^M = \frac{1/9}{2(1/4)} = 2/9 \approx 0.222.$$

(3) $f(x) = 3^x \quad \frac{d}{dx}f(x) = (\ln 3)3^x$

$$\text{arc length} = \int_0^1 \sqrt{1+\left[(\ln 3)3^x\right]^2}\,dx$$

$g(x) = \sqrt{1+(\ln 3)^2 3^{2x}}$ Simpson's rule:

$$AL \approx \frac{1/2}{3}\left(g(0) + 4g(1/2) + g(1)\right)$$

≈ 2.2547

(4) 

Method of disks

$$V = \int_0^\pi \pi(\sin x)^2\,dx$$

$$= \pi\int_0^\pi \tfrac{1}{2}(1-\cos 2x)\,dx$$

$$= \pi/2\left(x - \tfrac{1}{2}\sin 2x\right)\Big]_0^\pi$$

$$= \pi^2/2 \approx 4.9348$$

(5) 

Method of shells

$$V = \int_0^1 \frac{2\pi x\,dx}{6-x-x^2}$$

$$= 2\pi\int_0^1 \frac{-2/5}{x-2} + \frac{-3/5}{x+3}\,dx \quad \text{(partial fractions)}$$

$$= 2\pi\left[-\tfrac{2}{5}\ln|x-2| - \tfrac{3}{5}\ln|x+3|\right]\Big|_0^1$$

$$= \frac{-8\pi}{5}\ln 2 + \frac{6\pi}{5}\ln 3 \approx 0.6573$$

(6) $$\int \frac{dy}{y} = \int xe^x\,dx = xe^x - \int e^x\,dx = xe^x - e^x + C$$

$u = x \quad du = dx$

$dv = e^x dx \quad v = e^x$

$$\ln|y| = xe^x - e^x + C$$

$y = e^{xe^x - e^x + C}$ general

$1 = e^{0e^0 - e^0 + C} \Rightarrow C = 1$

$y = e^{xe^x - e^x + 1}$ specific

$y(2) = e^{2e^2 - e^2 + 1} = e^{e^2+1} \approx 4398.663$

(7) $$IF(x) = e^{\int \frac{2}{x}dx} = e^{2\ln|x|} = x^2$$

$$y = \frac{1}{x^2}\int x^2\left(\frac{\sin 3x}{x^2}\right)dx = \frac{1}{x^2}\int \sin 3x\,dx$$

$y = \frac{1}{x^2}\left(-\tfrac{1}{3}\cos x + C\right)$ general

$0 = y(\pi) = \frac{1}{\pi^2}\left(-\tfrac{1}{3}\cos\pi + C\right) \Rightarrow C = -1/3$

$y = \frac{1}{x^2}\left(-\tfrac{1}{3}\cos x - 1/3\right)$ specific

$$y(2\pi) = \frac{1}{(2\pi)^2}\left[-\tfrac{1}{3}\cos(2\pi) - \tfrac{1}{3}\right] = \frac{-1}{6\pi^2}$$

$\approx -.016887$

8% of you got 1+ grade higher (than your quizzes)

10% of you got 1/2-1 grade higher

50% of you got within 1/2 a grade

17% of you got 1/2-1 grade lower

14% of you got 1+ grade lower

total < 100% due to rounding