

# How to Factor 36 into Primes in Two Different Ways

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San Diego State University  
Mathematics Research Experience for Teachers

Professional development program, June 22-Aug. 14  
Math education project, pays \$7640 stipend

`www.sci.sdsu.edu/math-reu/index.html`



# The Primary Question

What is a prime number?



# The Primary Answers

- A number with no divisors
- A number whose only divisors are 1 and itself
- A number whose only integer divisors are 1 and itself
- A number whose only positive integer divisors are 1 and itself





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# Primarily Relevant Facts

- If a prime divides  $xy$ , then it must divide either  $x$  or  $y$ .
- Fundamental Theorem of Arithmetic:  
Every natural number has a factorization into primes, unique up to order.



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# Attack of the Jargon

- First step: Specify our set of numbers (need  $\times$ )
- A **unit** is a number that has a reciprocal in our set.  
A **nonunit** is everything else.
- Examples:  $\mathbb{N}$ ,  $\mathbb{Z}$ ,  $\mathbb{Q}$



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- A **prime** is a nonunit that cannot be factored into nonunits.
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# Review

- A **unit** has a reciprocal.
- A **prime** is a nonunit that cannot be factored into nonunits.



# Number Pluralism

$\mathbb{N}^{\geq 2} = \{2, 3, 4, 5, 6, \dots\}$  “plural” numbers



# An Odd Example

$$\mathbb{N}^{\text{odd}} = \{1, 3, 5, 7, 9, \dots\} \text{ "odd" numbers}$$



# Even Primes Can Have Fun

$\mathbb{N}^{\text{even}} = \{2, 4, 6, 8, 10, \dots\}$  “even” numbers

Primes: 2, 6, 10, 14, 18, 22, ... (singly even)

$$36 = 6 \times 6 = 2 \times 18$$



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# Many Primes

$\mathbb{N}^{\geq 3} = \{3, 4, 5, 6, 7, 8, \dots\}$  “many” numbers (Pirahã)

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# Super Primes

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# Super-Fundamental Theorem

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If a number can be factored into super-primes, that factorization is unique up to order and units.

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# Guide to Further Inquiry

- This topic is **factorization theory**.
- The “numbers” can be **domains, rings, semirings, monoids** or (most generally) **semigroups**.
- What we called “prime” is usually called **irreducible**.
- What we called “super-prime” is usually called **prime**.
- What we called “units” are usually called **units**.



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