HOW TO OBTAIN SYNTHETIC SEISMOGRAMS FROM PEER/SCEC TEST PROBLEMS FOR ANY SOURCE FUNCTION

An Informal Appendix to PEER Reports 1A01 and 1A02
S. M. Day (Version 1, August 2007)

The simulations UHS.1-2 and LOH.1-4 were done with a minimum-phase moment-rate function of form $M_0 S_{sim}(t)$, where

$$S_{sim}(t) = \frac{t}{T^2} e^{-t/T} \tag{1}$$

in which T is a characteristic source time. Then later, when analyzing and plotting the results, this source was deconvolved from the synthetics, and the result convolved with a Gaussian of form

$$S_{new}(t) = \left(\sqrt{2\pi}\sigma\right)^{-1} \exp\left[-(t-4\sigma)^2 / (2\sigma^2)\right] \tag{2}$$

where σ is the spread of the Gaussian and determines its bandwidth. The 4σ is present so that there will be negligible jump when this source is turned on at time zero (i.e., windowed by a step function $H(t)$).

The reason for this strategy is as follows: We run with the low-pass minimum phase source so that the output will be sufficiently smooth to make snapshot plots of velocity without additional filtering, since the snapshots are informative themselves and help identify errors in problem setup.

But then we want to look at time series in various pass bands, so we deconvolve the original source and put in various others for making comparisons in different bands, without having to re-run the simulations. The deconvolution is stable because it is done analytically, not numerically: the inverse of the source wavelet is a simple differentiation operator that can be applied analytically to any new source function (like the Gaussians) before it is convolved with the time series. That's why I chose the special form of the original source function, i.e., so that it was low frequency enough to make nice snapshots, but also had an analytical inverse wavelet. This turned out to be quite a flexible arrangement, because of the ability it gave us to experiment with different passbands for the time domain comparisons (without rerunning the simulations).

The analytic inverse of the source pulse is

$$S_{sim}^{-1}(t) = \left(1 + T\frac{d}{dt}\right)^2 \tag{3}$$

So, if you want to take out the original source and put in a new one, just convolve the original synthetic (raw output of the simulation) with

$$F(t) = \left(1 + T\frac{d}{dt}\right)^2 S_{new}(t) \tag{4}$$

Plug in the Gaussian (2) for S_{new} in (4), carry out the differentiation, and you get (ignoring the 4σ time offset)

$$F(t) = \left(\sqrt{2\pi}\sigma\right)^{-1} \exp\left[-(t-4\sigma)^2 / \left(2\sigma^2\right)\right] G(t) \tag{5}$$

where

$$G(t) = 1 - \frac{2T}{\sigma^2}t - \frac{T^2}{\sigma^2}\left(1 - \frac{t^2}{\sigma^2}\right) \tag{6}$$

Then filter (convolve) the raw synthetics with *F*, using (5) to define *F* (and replacing t with t-4σ to recover the delay), and you get the results plotted in the PEER reports. Values of T and σ are reported therein. The same procedure was used in comparisons of UHS.1 (and finite Q modifications of it) to get the results in the following paper by Day and Bradley (2001):

Day, S.M., and Bradley, C. (2001). Memory-efficient simulation of anelastic wave propagation, *Bull. Seism. Soc. Am.*, *Vol. 91*, 520-531.

Details leading to (3)

To derive (3), take the Laplace transform of (1)

$$\bar{S}_{sim}(s) = (1 + sT)^{-2} \tag{7}$$

and take its algebraic inverse

$$\bar{S}_{sim}^{-1}(s) = (1 + sT)^2 \tag{8}$$

Then invert the Laplace transform,

$$S_{sim}^{-1}(t) = \left(1 + T\frac{d}{dt}\right)^2 \tag{9}$$