HW 1, Astro Solutions

1) $F_g = \frac{MmG}{r^2}$; MmG is constant for both cases, r is not.

$R_e = 6378$ km ; $r = 6378 + 40 = 6418$

$F_{g1} = \frac{MmG}{R_e^2}$; $F_{g2} = \frac{MmG}{r^2}$

$\Delta F_g = \frac{\cancel{Mm}G}{r^2} \cdot \frac{R_e^2}{\cancel{MmG}} = \frac{R_e^2}{r^2}$; $\Delta F_g = \frac{6378^2}{6418^2} = 0.9875$

$\Delta m = 1 - 0.9875$; $\boxed{\Delta m = 0.0124 = 1.24\%}$

2) heading east (with rotation of the earth)

Gravity change (as above) $\Delta F_g = 1 - \frac{6378^2}{(6378+12)^2} = 0.375\%$.

Centrifugal force:

Centrifugal force on the ground: $F_c = 2\pi R_e \cdot C_R \cdot (1\,\text{day})$

C_R - rotation coefficient eastward (kinda) (see problem 5)

$F_c = 2\pi\, 6378 \cdot \frac{365.25}{(366.25)} \cdot \frac{1}{(86400)} = 0.463 \frac{km}{s} = 463 \frac{m}{s}$

So, while flying east, net velocity $V = 463 + 250 = 713 \frac{m}{s}$

(for west $V = 463 - 250 = 213$ m/s - against rotation of the planet)

So, centrifugal acceleration: $a_c = \frac{F_c}{m} = \frac{V^2}{r}$

$a_c' = \frac{713^2}{(6378+12)\cdot 1000} = 0.08 \frac{m}{s^2}$ (going east, while flying at altitude)

$a_c'' = \frac{463^2}{6378000} = 0.034 \frac{m}{s^2}$ (going east, while flying at altitude).

So, mass is 0.375% lighter due to altitude, and

$\frac{(0.08-0.034)}{9.8} = 0.47\%$ lighter due to velocity,

for total Δm of $(0.375+0.47)\% = \boxed{0.845\%}$

3) San Diego is 32° latitude from equator

So, $R = 6378 \cdot \cos(32°) = 5408.85$ km

$V_c = 2\pi R \cdot \frac{365.25}{366.25} \cdot \frac{1}{86400} = 0.3923 \frac{km}{s}$

$a_c = \frac{F_c}{m} = \frac{V^2}{r} = \frac{0.3923^2}{5408.85} = 0.028 \frac{m}{s^2}$

So, $\Delta m = \frac{0.028}{9.81} = 0.29\%$

4) $R_e^e = 6378$ km ;
$R_p^e = 6357$ km ; $\Delta R = 21$ km.

So, variation of earth radius with latitude is
$\partial R = 6378 - 21 \cdot \left[1 - \frac{\cos(2 \cdot \text{latitude})}{2}\right]$

Worst case scenario, is when $\boxed{\text{latitude} = 45°}$

5) In order to be weightless, $F_c = F_g$. (centrifugal force must counteract gravity)

Force of gravity at equator (at sea level) has $g = 9.81\ m/s^2$

So, $V = \sqrt{g\,r} = \sqrt{9.81 \cdot 6378000}$; $V = 7910\ m/s$

$V_{need} = 7910 - \underset{\uparrow}{465} = \boxed{7445\ m/s}$,
eastward motion of the equator

6) $a = g = 9.81\ m/s^2$
Velocity needed is $V = 7445\ m/s$
So $t = \frac{V}{a} = \frac{7445}{9.81} = 759$ sec needed to reach the velocity.

$V_{average} = \frac{7445 + 0}{2} = 3722.5\ m/s$

$d = V\,t = 3722.5 \cdot 759 = \boxed{2825.1\ km}$

7) Velocity required: $V = 7445\ m/s$ (from problem 6)

But it took a distance of 2825.1 km and acceleration of 1g to reach this velocity, so if distance is decreased to just 1 km, the acceleration must become $\boxed{2825\ 1g}$

8) $R_o = 6378 + 1000 = 7378$ km

$V = \sqrt{\frac{\mu}{R}}$; $a_c = \frac{V^2}{R}$; $a_c R = \frac{\mu}{R}$; $\boxed{a_c = \frac{\mu}{R^2}}$

$a_c = 398600 \frac{km^3}{s^2} \cdot \frac{1}{(7378)^2\ km^2}$ $a_c = 7.32 \frac{m}{s^2}$

Displacing an object 100 m requires $\frac{100}{7378000}$ of that acceleration or $a_r = 0.000099$ m/s^2 required

from $E = mc^2$; the momentum of light is $p = mc = E/c$
Assume perfectly reflecting sail => $\boxed{P = 2E/C}$
The energy density of sunlight (1 au from Sun) = 1373 W/m^2

So pressure on perfectly reflecting mylar is

$$P = 2 \cdot 1373 \frac{J}{s \cdot m^2} \cdot \frac{1}{c} = 1373 \cdot kg \frac{m^2}{s^2} \cdot \frac{1}{s\,m^2} \cdot \frac{1}{(300000000)} \frac{s}{m}$$

$P = 9.15 \cdot 10^{-6}$ N/m^2

$$\text{Areal density} = 9.15 \cdot 10^{-6}\ kg \cdot \frac{m}{s^2} \cdot \frac{1}{m^2} \cdot \frac{1}{0.000099} \frac{s^2}{m} = 0.0924 \frac{kg}{m^2}$$

$$\text{Area/mass ratio} = \frac{1}{0.0924} = \boxed{10.82 \frac{m^2}{kg}}$$

$$\text{Density } \rho = 1.4 \frac{g}{cm^3} = 1400 \frac{kg}{m^3}$$

$$t = \left(10.82 \cdot 1400 \frac{m^2}{kg} \cdot \frac{kg}{m^3}\right)^{-1} \quad \boxed{t = 0.000066\ m}$$

3) $$V = \sqrt{\frac{\mu}{R}} = \sqrt{398600 \frac{km^3}{s^2} \cdot \frac{1}{(6378 + 350)\,km}}$$

$V = 7.7$ m/s (main body velocity)

$$V_p = 7.697 \cdot \frac{6708}{6728} = 7.674\ km/s \text{ (payload velocity). } (R = 6708\ km)$$

use eq 3.14: $V = \sqrt{\frac{2\mu}{r} - \frac{\mu}{a}}$, where a is semi-major axis.

solve for a: $$7.674^2 = 2\,\frac{398600}{6708} - \frac{398600}{a}$$

$\boxed{a = 6648.88 \text{ km}}$

Perigee $r_p = 2a - r_a = 2 \cdot 6648.88 - 6708$; $\boxed{r_p = 6589.76 \text{ km}}$

Period $P = 2\pi\sqrt{\frac{r^3}{\mu}}$

Period for Delta : $P = 2\pi\sqrt{\frac{6728^3}{\mu}} = 5492.12 \text{ s}$

Period for Payload $P = 2\pi\sqrt{\frac{6648.88^3}{\mu}} = 5395.52 \text{ s}$

$\boxed{\Delta P = 96.6 \text{ s}}$

10. Why are tides high about twice per month?

It is about a month between one new moon and the next. During that period, the relative orientation of sun and moon go from conjunction, to quadrature, to opposition, to quadrature, and then to conjunction. Since tidal forces have bulges on both near and far sides, the solar and lunar effects add up at both conjunction and opposition, while at quadrature, they tend to cancel each other out. Since the moon has ~2.2X the effect of the sun, the relative heights of the tides can range from roughly 2.2+1 to 2.2-1, or 3.2 to 1.2, or roughly 2.7 to 1. The high tides are called "spring tides" and the low tides are called "neap tides." Note that the moon's orbit has an eccentricity of 0.055, while the earth's orbit around the sun has an eccentricity of only 0.0167. Tidal effects scale with inverse cube of distance, so the time in the year has only a 3*1.67% effect on the solar component of tides, or about 5% above or below the mean. But the lunar component can vary by +/-16%. When you hear about particularly high or low tides, it's because it's a new moon or full moon near the time of lunar perigee.