

**Consider the function  $f(x) = x^3 + 6x^2 - 15x + k$ . Find all values of  $k$  such that the absolute maximum value and the absolute minimum value of  $f$  on the interval  $[-10, 2]$  have the same absolute value.**

Given that the function is a simple polynomial we know that it is continuous and defined in all  $x$ . In finding the absolute maximum and absolute minimum of the cubic function we only need to check the  $f(x)$  values at the critical numbers and the endpoints.

Remembering that  $k$  is a constant we derive the function and then solve for  $x$ .

$$f' = 3x^2 + 6x - 15 \quad (\text{derivative})$$

$$0 = 3x^2 + 6x - 15 \quad (\text{solving for } x)$$

$$0 = 3(x - 1)(x + 5)$$

$$x = 1 \text{ and } x = -5 \quad (\text{our critical numbers})$$

We plug these values in our original function, letting  $k = 0$ , and get

$$f(1) = -8 \text{ and } f(-5) = 100.$$

We now check the endpoints of our interval  $[-10, 2]$  in  $f(x)$ , again letting  $k = 0$ .

$$f(-10) = -250 \text{ and } f(2) = 2.$$

Then our absolute maximum is at  $x = -5$  and our absolute minimum is at  $x = -10$ .

Since  $k$  is just an added constant its only role is to shift the graph of the function up or down. This means that our absolute maximum and absolute minimum will always be at  $x = -5$  and  $x = -10$  respectively regardless of the value of  $k$ .

We want the absolute value of  $f(-5)$  and  $f(-10)$  to be the same so their distance from the  $x$ -axis has to be the same. Then we should find the distance between the two and divide it in half.

$$f(-5) = 100 \text{ and } f(-10) = -250$$

$$100 + 250 = 350 \quad (\text{distance between the two})$$

$$350/2 = 175 \quad (\text{half of their distance})$$

So we want  $f(-5) = 175$  and  $f(-10) = -175$ . Since  $f(-5) = 100$  we let  $k = 75$ . This gives us  $f(-5) = 175$  and  $f(-10) = -175$ . Since  $k$  only shifts the graph, there are no other values of  $k$  such that the absolute value of the absolute maximum and absolute minimum of the function  $f(x) = x^3 + 6x^2 - 15x + k$  are the same. Any  $k > 75$  will make the absolute value of the absolute maximum greater than the absolute value of the absolute minimum and any  $k < 75$  would make the absolute value of the absolute minimum greater than the absolute value of the absolute maximum. Hence  $k = 75$  is our only answer.