1. The sum $S = -\sum_{i=1}^{n} p_i \ln(p_i)$ is a measure of the amount of randomness in a probability distribution $P = (p_1, p_2, ..., p_n)$. Find the most random distribution over three categories, such that $p_2 = 2p_3$, i.e., find $P = (p_1, p_2, p_3)$ which maximizes S subject to $\sum_{i=1}^{3} p_i = 1$ and $p_2 = 2p_3$.

2. Verify that the function

$$f_0(X) = f_0(x,y,z) = x^2 + y^2 + z^2$$

subject to

$$f_1(X) = f_1(x,y,z) = x^2 - 4y = 0$$

and

$$f_2(X) = f_2(x,y,z) = x - y - 1 = 0$$

has a minimum at $X_0=(2,1,0)$ but that f_0' is not expressible as $\lambda_1 f_1' + \lambda_2 f_2'$ at X_0. What happened?

3. A fund manager is choosing a portfolio of stocks in which to invest. The return per dollar investment in stock j has been on the average m_j, its variance has been $s_{jj} = s_j^2$ and its covariance with stock k has been s_{jk}. The covariance matrix $S = [s_{jk}]$ is symmetric and positive definite. Let x_j be the amount invested in stock j and let $X=(x_1, \ldots, x_n)$. Then X has expected return $m = \Sigma_j x_j m_j$ and variance $s^2 = X^T S X$.

a. Formulate the general problem of maximizing the expected return on a dollar with a given variance or, equivalently, minimize the variance for a given mean return.

b. What is the minimum variance portfolio which will guarantee a mean return of 0.13 per dollar invested in three stocks with the following data:

$$m_1=0.15,\ m_2=0.10,\ m_3=0.09,$$

$$S = \begin{bmatrix} 0.0625 & 0.0075 & -0.001 \\ 0.0075 & 0.0225 & 0.0010 \\ -0.001 & 0.0010 & 0.0012 \end{bmatrix}$$

4 A. Find (x,y,z) which maximizes f(x,y,z)=xyz subject to the constraint

$$c(x,y,z) = 2x+3y+z = 1$$

where x, y, and z are positive. Also find the value of the Lagrange multiplier.

Hint: form ratios.

4B. Find (x,y,z) which minimizes f(x,y,z)=2x+3y+z subject to the constraint

$$c(x,y,z) = xyz = 1/162$$

where x, y, and z are positive. Also find the value of the Lagrange multiplier.

4C. Relate the two solutions in A and B above; pay special attention to the respective values of the Lagrange multipliers.