1. Find the area of the largest rectangle with sides parallel to the axes that can be inscribed in the ellipse $x^2 + y^2/4 = 1$.

2. If the sum of two positive numbers is 30, what is the maximum value of their product?

3. Find a necessary condition for the point on the surface $f(x,y,z)=0$ which lies nearest to a given point $(x_0,y_0,z_0)$. Show that if $(x^*,y^*,z^*)$ satisfies your condition, then the vector from $(x^*,y^*,z^*)$ to $(x_0,y_0,z_0)$ is perpendicular to the surface $f=0$ at $(x^*,y^*,z^*)$.

4. The sum $S = - \sum_{i=1}^{n} p_i \ln(p_i)$ is a measure of the amount of randomness in a probability distribution $P = (p_1, p_2, ..., p_n)$. Find the most random distribution over three categories, i.e., find $P = (p_1, p_2, p_3)$ which maximizes $S$ subject to $\sum_{i=1}^{n} p_i = 1$. 