

1. Find the area of the largest rectangle with sides parallel to the axes that can be inscribed in the ellipse $x^2+y^2/4=1$.
2. If the sum of two positive numbers is 30, what is the maximum value of their product?
3. Find a necessary condition for the point on the surface $f(x,y,z)=0$ which lies nearest to a given point (x_0,y_0,z_0) . Show that if (x^*,y^*,z^*) satisfies your condition, then the vector from (x^*,y^*,z^*) to (x_0,y_0,z_0) is perpendicular to the surface $f=0$ at (x^*,y^*,z^*) .
4. The sum $S = - \sum_{i=1}^n p_i \ln(p_i)$ is a measure of the amount of randomness in a probability distribution $P = (p_1, p_2, \dots, p_n)$. Find the most random distribution over three categories, i.e., find $P = (p_1, p_2, p_3)$ which maximizes S subject to $\sum_{i=1}^n p_i = 1$.