1. Find the quadratic approximation to each of the following functions at the indicated base points and use this quadratic map to approximate the value of the function at the nearby points indicated.

Table 1: default

function	base point	nearby point
$f(x,y) = 3xy + 17x^2$	$(1,2)$	$(2,1)$
$\gamma(t) = (t^3, sin(\pi t))$	2	3
$g(x,y) = \left(\sqrt{x^2+y^2}, arctan(y/x)\right)$	$(1,1)$	$(1,0)$
$h(x) = x^2$	2	3

2a. Using the first order necessary conditions, find a minimum point of the function

$$f(x,y,z) = 2x^2 + xy + y^2 + yz + z^2 - 6x - 7y - 8z + 9 \tag{1}$$

2b. Verify that the point is a local minimum by verifying that the second order sufficiency conditions hold.

2c. Prove that the point is a global minimum point.

3a. A rocket ship is traversing the trajectory

$$\gamma(t) = (t^2, 2t+1, t-1)' \tag{2}$$

where the coordinates of γ are relative to a coordinate system with the sun at the center. Find the position of closest approach of the rocket ship to the sun.

3b. If the radiation from the sun is given by

$$f(x,y,z) = \frac{1}{x^2+y^2+z^2}, \tag{3}$$

argue that this function is maximum at the point you found in part (a).