

Section	2.2	2.3	3.1	3.2	3.3	3.4	3.5
Total questions	06	09	06	04	03	03	04
Total x/35							①

2.2  
(2a) Evaluate from inspection

$$\begin{vmatrix} 3 & -17 & 4 \\ 0 & 5 & 1 \\ 0 & 0 & -2 \end{vmatrix} \Rightarrow \cancel{30} \quad 3 \times 5 \times (-2)$$

$$\boxed{\det(A) = -30}$$

(3a)

$$\begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -5 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} \Rightarrow 1 \begin{vmatrix} 1 & 0 & 0 \\ 0 & -5 & 1 \\ 0 & 0 & 1 \end{vmatrix} \Rightarrow 1 \times 1 \times (-5) \times 1$$

$$\boxed{\det(A) = -5}$$

(5)

$$A = \begin{bmatrix} 0 & 3 & 1 \\ 1 & 1 & 2 \\ 3 & 2 & 4 \end{bmatrix} \quad \det(A) = \begin{vmatrix} 0 & 3 & 1 \\ 1 & 1 & 2 \\ 3 & 2 & 4 \end{vmatrix} = - \begin{vmatrix} 1 & 1 & 2 \\ 0 & 3 & 1 \\ 3 & 2 & 4 \end{vmatrix} \quad (R_1 \leftrightarrow R_2)$$

$$\begin{vmatrix} 1 & 1 & 2 \\ 0 & 3 & 1 \\ 0 & -1 & -2 \end{vmatrix} \xrightarrow{R_3 \leftarrow R_1 \times (-3)} \begin{vmatrix} 1 & 1 & 2 \\ 0 & 3 & 1 \\ 0 & 0 & -5/3 \end{vmatrix} \xrightarrow{R_3 \leftarrow R_2 \times (1/3)}$$

$$\rightarrow \frac{1}{3} \begin{vmatrix} 1 & 1 & 2 \\ 0 & 3 & 1 \\ 0 & 0 & 5 \end{vmatrix} \xrightarrow{\text{common factor } -1/3} = \frac{1}{3} (1 \times 3 \times 5) = 5$$

$$\boxed{\det(A) = 5}$$

(8)

$$A = \begin{bmatrix} 1 & -2 & 3 & 1 \\ 5 & -9 & 6 & 3 \\ -1 & 2 & -6 & -2 \\ 2 & 8 & 6 & 1 \end{bmatrix} \quad \det(A) = \begin{vmatrix} 1 & -2 & 3 & 1 \\ 5 & -9 & 6 & 3 \\ -1 & 2 & -6 & -2 \\ 2 & 8 & 6 & 1 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & -2 & 3 & 1 \\ 0 & 1 & -9 & -2 \\ 0 & 0 & -3 & -1 \\ 0 & 12 & 0 & -1 \end{vmatrix} \begin{array}{l} R_2 \leftarrow R_1 \times (-5) \\ R_3 \leftarrow R_1 \times 6 \\ R_4 \leftarrow R_1 \times (-2) \end{array} \rightarrow \begin{vmatrix} 1 & -2 & 3 & 1 \\ 0 & 1 & -9 & -2 \\ 0 & 0 & -3 & -1 \\ 0 & 0 & 108 & 23 \end{vmatrix} \begin{array}{l} R_4 \leftarrow R_2 \times 12 \end{array}$$

$$\rightarrow \begin{vmatrix} 1 & -2 & 3 & 1 \\ 0 & 1 & -9 & -2 \\ 0 & 0 & -3 & -1 \\ 0 & 0 & 0 & -13 \end{vmatrix} \begin{array}{l} R_4 \leftarrow R_3 \times 3 \end{array} = 1 \times 1 \times (-3) \times (-13) = +39$$

$$\boxed{\det(A) = +39}$$

(12)

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = -6$$

(b)

$$\begin{vmatrix} 3a & 3b & 3c \\ -d & -e & -f \\ 4g & 4h & 4i \end{vmatrix} = (-12) \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = (-12) \times (-6) = 72$$

$\begin{array}{l} 3 \text{ km } R_1 \\ -1 \text{ km } R_2 \\ 4 \text{ km } R_3 \end{array}$

$$\boxed{\det(A) = 72}$$

(d)

$$\begin{vmatrix} -3a & -3b & -3c \\ d & e & f \\ g-4d & h-4e & i-4f \end{vmatrix} = -3 \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} \begin{array}{l} R_1 \leftarrow R_1 \times (-3) \\ R_3 \leftarrow R_3 - 4R_1 \end{array}$$

$$= (-3) \times (-6) = 18$$

$$\boxed{\det(A) = 18}$$

(2)

2.3

$$(5) \quad A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \quad \det(A) = -7.$$

(a)  $\det(3A)$

$$\det(kA) = k^n (\det A) \quad \text{if } A \text{ is } n \times n$$

$$\det(3A) = 3^3 (-7) = 27 \times (-7) = -189$$

$$\boxed{\det(3A) = -189}$$

(b)  $\det(A^{-1}) = \frac{1}{\det(A)} = -\frac{1}{7}$

$$\boxed{\det(A^{-1}) = -1/7}$$

(c)  $\det(2A^{-1}) = 2^3 \times \frac{1}{\det(A)} = \frac{8}{-7} = -8/7$

$$\boxed{\det(2A^{-1}) = -8/7}$$

(14)  $(\lambda I - A)x = 0$

(a)  $\left. \begin{matrix} x_1 + 2x_2 = \lambda x_1 \\ 2x_1 + x_2 = \lambda x_2 \end{matrix} \right\} \Rightarrow$  From  $Ax = \lambda x$

$$\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \lambda \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\boxed{\begin{bmatrix} \lambda - 1 & -2 \\ -2 & \lambda - 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}}$$

(b)  $\left. \begin{matrix} 2x_1 + 3x_2 = \lambda x_1 \\ 4x_1 + 3x_2 = \lambda x_2 \end{matrix} \right\} \Rightarrow$

$$\begin{bmatrix} 2 & 3 \\ 4 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \lambda \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} - \begin{bmatrix} 2 & 3 \\ 4 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\boxed{\begin{bmatrix} \lambda - 2 & -3 \\ -4 & \lambda - 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}}$$

(15) (i) Characteristic Equation.

(a)  $(\lambda I - A) = 0$ .

$$\begin{vmatrix} \lambda - 1 & -2 \\ -2 & \lambda - 1 \end{vmatrix} = 0 \quad \text{OR} \quad (\lambda - 1)^2 - 4 = 0$$

(b)  $\begin{vmatrix} \lambda - 2 & -3 \\ -4 & \lambda - 3 \end{vmatrix} = 0$  OR  $(\lambda - 2)(\lambda - 3) - 12 = 0$   
 $\lambda^2 - 5\lambda - 6 = 0$ .

(ii) Eigen Values:

(a)  $(\lambda - 1)^2 = 4$

$$\lambda - 1 = \pm 2$$

$$\boxed{\lambda = 3 \text{ OR } \lambda = -1}$$

(b)  $(\lambda - 2)(\lambda - 3) - 12 = 0$

$$\Rightarrow \lambda^2 - 5\lambda - 6 = 0$$

$$\Rightarrow \lambda^2 - 6\lambda + \lambda - 6 = 0$$

$$\Rightarrow \lambda(\lambda - 6) + 1(\lambda - 6) = 0$$

$$\Rightarrow (\lambda - 6)(\lambda + 1) = 0$$

$$\boxed{\lambda = 6 \text{ OR } \lambda = -1}$$

(3)

### 3.1

(3a) Component of the vectors. initial point  $P_1$  terminal  $P_2$

$P_1(4, 8), P_2(3, 7)$

$$\vec{P_1P_2} = (3-4, 7-8) = (-1, -1)$$

$$\boxed{\vec{P_1P_2} = (-1, -1)}$$

(4a) nonzero vector  $u$ . initial point  $P(-1, 3, -5)$  such that  $u$  has the same direction as  $v = (6, 7, -3)$

~~vector~~ when  $u$  &  $v$  have the same direction

$u = k(6, 7, -3)$  direction vector =  $(6, 7, -3)$

$$\therefore \boxed{v = (-1, 3, -5) + t(6, 7, -3)}$$

(6b)  $u = (-3, 1, 2), v = (4, 0, -8), w = (6, 1, -4)$

But  $2v = (-12, 6, -16)$

$$\boxed{6u + 2v = (-10, 6, -4)}$$

(10) Find scalars  $c_1, c_2$  &  $c_3$  such that

$$c_1(1, 2, 0) + c_2(2, 1, 1) + c_3(0, 3, 1) = (0, 0, 0)$$

$$c_1 + 2c_2 + 0c_3 = 0 \quad \text{--- (i)}$$

$$2c_1 + c_2 + 3c_3 = 0 \quad \text{--- (ii)}$$

$$0c_1 + c_2 + c_3 = 0 \quad \text{--- (iii)}$$

(i)  $\times$  (2) + (ii)  $\quad -3c_2 + 3c_3 = 0$

$\quad \quad \quad \& \quad c_2 + c_3 = 0$

$$\therefore \boxed{c_1 = c_2 = c_3 = 0}$$

(11) (a)  $P(2, 3, -2)$  &  $Q(7, -4, 1)$

midpoint of segment connecting  $PQ$

$$\left( \frac{7+2}{2}, \frac{-4+3}{2}, \frac{1+(-2)}{2} \right) = \left( \frac{9}{2}, -\frac{1}{2}, -\frac{1}{2} \right)$$

$$\boxed{\left( \frac{9}{2}, -\frac{1}{2}, -\frac{1}{2} \right)}$$

(13)  $P(1, 3, 7)$  &  $Q(4, 0, 6)$  is midpoint connecting  
 $P$  &  $Q$

$$(4, 0, 6) = \left( \frac{1+2x+1}{2}, \frac{y+3}{2}, \frac{7+z}{2} \right)$$

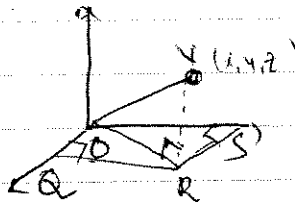
$$\begin{aligned} 2+2x &= 8 & y+3 &= 0 & 7+z &= -12 \\ x &= 3 & y &= -3 & z &= -19 \end{aligned}$$

$$\boxed{Q(3, -3, -5)} \quad \boxed{Q(7, -3, -19)}$$

3.2

i) Norm of  $v$

(d)  $\vec{v} = (2, 2, 2)$



$$\|v\| = \sqrt{(2-0)^2 + (2-0)^2 + (2-0)^2} = \sqrt{4+4+4} = \sqrt{12} = 2\sqrt{3}$$

$$\boxed{\|v\| = 2\sqrt{3}}$$

2(a) Distance between  $P_1$  &  $P_2$

$$P_1(3, 4), \quad P_2(5, 7)$$

$$\vec{P_1P_2} = (2, 3)$$

$$|P_1P_2| = \sqrt{4+9} = \sqrt{13}$$

$$\boxed{|P_1P_2| = \sqrt{13}}$$

5(a)  $u = (2, 0, 4)$  &  $v = (1, 3, -6)$

$$k(u) + l(v) = (5, 9, -14)$$

$$k(2, 0, 4) + l(1, 3, -6) = (5, 9, -14)$$

$$2k + l = 5$$

$$3l = 9 \Rightarrow l = 3$$

$$4k - 6l = -14$$

$$\Rightarrow l = 3$$

$$\therefore 2k + 3 = 5 \Rightarrow 2k = 2 \Rightarrow k = 1$$

$$\boxed{k=1 \text{ \& } l=3 \text{ such that } k\mathbf{u} + l\mathbf{v} = (5, 9, -14)}$$

(9)(b) if  $\mathbf{v}$  is non zero vector then  $\frac{1}{\|\mathbf{v}\|} \mathbf{v}$  is

a unit vector.

Use this next to find vector that has the same direction as the vector  $(3, 4)$

$$\|\mathbf{v}\| = \sqrt{9+16} = \sqrt{25} = 5$$

$$\therefore (u_1, u_2) = (3/5, 4/5)$$

$$\boxed{\text{Unit vector in direction of } (3, 4) = (3/5, 4/5)}$$

3.3:

(2a)  $\mathbf{u} = (2, 3), \mathbf{v} = (5, -7)$

find the cosine of the angle  $\theta$  between  $\mathbf{u}$  &  $\mathbf{v}$ .  
 $\theta$  is the angle between  $\mathbf{u}$  &  $\mathbf{v}$ . then

$$\cos \theta = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\| \|\mathbf{v}\|} = \frac{(2, 3) \cdot (5, -7)}{\sqrt{4+9} \cdot \sqrt{25+49}}$$

$$= \frac{10 - 21}{\sqrt{13} \times \sqrt{74}} = \frac{-11}{\sqrt{74 \times 13}} = \frac{-11}{31.016}$$

$$\therefore \boxed{\cos \theta \approx -0.355}$$

(16) (a)  $p = (2, k)$  &  $q = (3, 5)$  find  $k$  such that

(a)  $p$  &  $q$  are parallel

if  $p$  &  $q$  are parallel then

$$p = nq$$

$$\therefore 2 = n \cdot 3 \quad \& \quad k = n \cdot 5$$

$$\Rightarrow n = \frac{2}{3} \quad \Rightarrow \quad k = \frac{2}{3} \times 5 \quad \therefore k$$

$$\boxed{k = 10/3}$$

(b)  $p$  &  $q$  are orthogonal

$$p \cdot q = 6 + 5k$$

For orthogonal solutions  $p \cdot q = 0$

$$\therefore 6 + 5k = 0$$

$$\Rightarrow \boxed{k = -6/5}$$

3.4:

(a)  $u = (3, 2, 4)$ ,  $v = (0, 2, -3)$  &  $w = (2, 6, 7)$

(a)  $v \times w$

$$v \times w = \begin{vmatrix} i & j & k \\ 0 & 2 & -3 \\ 2 & 6 & 7 \end{vmatrix}$$

$$= (14 + 18)i - (6)j + (-4)k$$

$$= 22i - 6j - 4k$$

$$\therefore \boxed{(v \times w) = (22, -6, -4)}$$

(a) If  $2 \cdot (v \times w) = 3$  Find

(a)  $2 \cdot (w \times v)$

$$w \times v = \begin{vmatrix} i & j & k \\ w_1 & w_2 & w_3 \\ v_1 & v_2 & v_3 \end{vmatrix} \quad \& \quad v \times w = \begin{vmatrix} i & j & k \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix}$$

Interchanging 2nd & 3rd row we can get one from another.

(5)

$$\begin{aligned} \therefore (v \times w) &= - (w \times v) \\ \Rightarrow 2 \cdot (v \times w) &= -4(w \times v) \\ \Rightarrow 4(w \times v) &= -3 \end{aligned}$$

$$\boxed{4(w \times v) = -3}$$

(11) (b)  $u = (5, -2, 1)$ ,  $v = (4, -1, 1)$  &  $w = (1, -1, 0)$

Determine whether  $u, v$  &  $w$  lie in the same plane ~~where~~ when positional so that their initial points coincide.

taking determinant value of these 3 points

$$\begin{vmatrix} 5 & -2 & 1 \\ 4 & -1 & 1 \\ 1 & -1 & 0 \end{vmatrix} = 5(0+1) - (-2)(0+1) + (-4+1) \\ = 5 - 2 - 3 \\ = 0$$

The absolute value of the determinant is zero, so the volume of the parallelepiped whose sides are the vectors  $u, v$  &  $w$  is equal to 0.

$$\boxed{u, v \text{ & } w \text{ lie in the same plane.}}$$

3.5.

(1) (a)  $P(-1, 3, -2)$ ;  $n = (-2, 1, -1)$

Find a normal form of the equation of the plane passing through  $P$  & having  $n$  as a normal. For given  $P$  &  $n$ , point normal form is given by

$$a(x-x_0) + b(y-y_0) + c(z-z_0) = 0$$

$$(-2)(x-(-1)) + (1)(y-3) + (-1)(z-(-2)) = 0$$

$$\boxed{-2(x+1) + (y-3) - (z+2) = 0}$$

(8) Determine whether the line and plane are perpendicular  
(b)  $x = 2+t, y = 1-t, z = 5+3t$ ;  $6x + 6y - 7z = 0$

$$(x, y, z) = (2, 1, 5) + t(1, -1, 3)$$

which is parallel to vector  $v(1, -1, 3)$

plane ~~line~~  $6x + 6y + 0z = 7$   
 $n(6, 6, 0)$

here we are not getting any relationship between  $n$  &  $v$ . such that  
 $n \neq kv$  for any  $k \in \mathbb{R}$

∴ Line & plane are not perpendicular

(10) Find parametric equation for the line passing through the given points.

(a)  $(5, -2, 4), (7, 2, -4)$

We are seeking parametric equation for line  $l$  which is passing through both points  $P_1$  &  $P_2$

$$\vec{P_1P_2} = (2, 4, -8)$$

Now selecting first point (any one for the given)

$$(x-5, y+2, z-4) = t(2, 4, -8)$$

$$\therefore \boxed{x = 5 + 2t, y = -2 + 4t, z = 4 - 8t} \quad (-\infty < t < \infty)$$

(21) Find an equation for the plane that passes through the point  $(3, -6, 7)$  & parallel to the plane  $5x - 2y + z - 5 = 0$

⇒ parallel planes have same normal

∴ Normal to plane  $5x - 2y + z - 5 = 0$  is  $n = (5, -2, 1)$

Now taking ~~equation~~ ~~to find~~ considering the given point

$$(5, -2, 1) \cdot (x-3, y+6, z-7) = 0$$

$$5x - 15 - 2y - 12 + z - 7 = 0$$

$$\boxed{5x - 2y + z - 34 = 0}$$