

①

1.6.

1)

$$x_1 + x_2 = 2$$

$$5x_1 + 6x_2 = 9$$

$$Ax = b$$

$$A = \begin{bmatrix} 1 & 1 \\ 5 & 6 \end{bmatrix}, \quad x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \quad b = \begin{bmatrix} 2 \\ 9 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 6 & -1 \\ -5 & 1 \end{bmatrix}$$

$$x = A^{-1}B$$

$$= \begin{bmatrix} 6 & -1 \\ -5 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 9 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$$

$$\boxed{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}}$$

5)

$$x + y + z = 5$$

$$x + y - 4z = 10$$

$$-4x + y + z = 0$$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & -4 \\ -4 & 1 & 1 \end{bmatrix}, \quad x = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad b = \begin{bmatrix} 5 \\ 10 \\ 0 \end{bmatrix}$$

$$Ax = B$$

$$x = A^{-1}B$$

$$= \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1/5 & 0 & -1/5 \\ 3/5 & 1/5 & 1/5 \\ 1/5 & -1/5 & 0 \end{bmatrix} \begin{bmatrix} 5 \\ 10 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 5 \\ -1 \end{bmatrix}$$

$$\boxed{\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 5 \\ -1 \end{bmatrix}}$$

16)

$$\begin{aligned} 6x_1 - 4x_2 &= b_1 \\ 3x_1 - 2x_2 &= b_2 \end{aligned}$$

augmented matrix $\begin{bmatrix} 6 & -4 & b_1 \\ 3 & -2 & b_2 \end{bmatrix}$

$$\begin{bmatrix} 0 & 0 & b_1 - 2b_2 \\ 3 & -2 & b_2 \end{bmatrix}$$

$$\boxed{b_1 - 2b_2 = 0}$$

17)

$$\begin{aligned} x_1 - 2x_2 + 5x_3 &= b_1 \\ 4x_1 - 5x_2 + 8x_3 &= b_2 \\ -3x_1 + 3x_2 - 3x_3 &= b_3 \end{aligned}$$

Augmented matrix $\begin{bmatrix} 1 & -2 & 5 & b_1 \\ 4 & -5 & 8 & b_2 \\ -3 & 3 & -3 & b_3 \end{bmatrix}$

$$\begin{bmatrix} 1 & -2 & 5 & b_1 \\ 0 & 0 & 0 & b_2 - b_1 + b_3 \\ -3 & 3 & -3 & b_3 \end{bmatrix}$$

$$R_1 \times (-1) \rightarrow R_2$$

$$R_3 \times 1 \rightarrow R_2$$

$$\boxed{-b_1 + b_2 + b_3 = 0}$$

20b)

$$A = \begin{bmatrix} 2 & 1 & 2 \\ 2 & 2 & -2 \\ 3 & 1 & 1 \end{bmatrix}$$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

②

$$Ax = 4x$$

$$\Rightarrow (A - 4I)x = 0$$

$$\Rightarrow x \neq 0$$

$$\begin{bmatrix} 2 & 1 & 2 \\ 2 & 2 & -2 \\ 3 & 1 & 1 \end{bmatrix} + \begin{bmatrix} -4 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & -4 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 2 \\ 2 & -2 & 2 \\ 3 & 1 & -3 \end{bmatrix}$$

$$\begin{bmatrix} 0 & -3 & 0 \\ 2 & -2 & 2 \\ 3 & 1 & -3 \end{bmatrix} \xrightarrow{R_1 \leftrightarrow R_2} \begin{bmatrix} 2 & -2 & 2 \\ 0 & -3 & 0 \\ 3 & 1 & -3 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & 0 & -2 \\ 0 & -3 & 0 \\ 3 & 0 & -3 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 0 & 1 & 0 \\ 2 & 0 & -2 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\therefore \begin{bmatrix} 0 & 1 & 0 \\ 2 & 0 & -2 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$\boxed{\begin{array}{l} x_2 = 0 \\ x_1 = x_3 \end{array}}$$

$$21) \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 0 \\ 0 & 2 & -1 \end{bmatrix} X = \begin{bmatrix} 2 & -1 & 5 & 7 & 8 \\ 4 & 0 & -3 & 0 & 1 \\ 3 & 5 & -7 & 2 & 1 \end{bmatrix}$$

A

B

$$A = 3 \times 3$$

$$B = 3 \times 5$$

$$\therefore X = 3 \times 5$$

$$X = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 3 & 0 \\ 0 & 2 & -1 \end{bmatrix}^{-1} \begin{bmatrix} 2 & -1 & 5 & 7 & 8 \\ 4 & 0 & -3 & 0 & 1 \\ 8 & 5 & -7 & 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -1 & 3 \\ -2 & 1 & -2 \\ -4 & 2 & -5 \end{bmatrix} \begin{bmatrix} 2 & -1 & 5 & 7 & 8 \\ 4 & 0 & -3 & 0 & 1 \\ 8 & 5 & -7 & 2 & 1 \end{bmatrix}$$

$$X = \begin{bmatrix} 11 & 12 & -3 & 27 & 26 \\ -6 & -8 & 1 & -18 & -17 \\ -15 & -21 & 9 & -38 & -35 \end{bmatrix}$$

③

1.7

$$(2) (a) \begin{bmatrix} 3 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ -4 & 1 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 6 & 3 \\ 4 & -1 \\ 4 & 10 \end{bmatrix}$$

$$(b) \begin{bmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 4 \end{bmatrix} \begin{bmatrix} 4 & -1 & 3 \\ 1 & 2 & 0 \\ -5 & 1 & -2 \end{bmatrix} \begin{bmatrix} -3 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} -24 & -10 & 12 \\ 3 & -10 & 0 \\ 60 & 20 & -16 \end{bmatrix}$$

$$\begin{bmatrix} 8 & -2 & 6 \\ -1 & -2 & 0 \\ -20 & 4 & -8 \end{bmatrix} \begin{bmatrix} -3 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} -24 & -10 & 12 \\ 3 & -10 & 0 \\ 60 & 20 & -16 \end{bmatrix}$$

(4) (a) $\begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix}$ not symmetric

(b) $\begin{bmatrix} 3 & 4 \\ 4 & 0 \end{bmatrix}$ symmetric

(c) $\begin{bmatrix} 2 & -1 & 3 \\ 1 & 5 & 1 \\ 3 & 10 & 7 \end{bmatrix}$ symmetric

(d) $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 2 & 0 \\ 3 & 0 & 0 \end{bmatrix}$ not symmetric

(6) $A = \begin{bmatrix} 2 & a-2b+2c & 2a+b+c \\ 3 & 5 & a+c \\ 0 & -2 & 7 \end{bmatrix}$

$$\begin{aligned} a-2b+2c &= 3 \\ 2a+b+c &= 0 \Rightarrow b+c = -2a \\ a+c &= -2 \Rightarrow c = -2-a \\ 2a+b-2-a &= 0 \Rightarrow a+b = 2 \\ a-2b-4-2a &= 3 \Rightarrow -a-2b = 7 \end{aligned}$$

$$\begin{aligned} -b &= 69 & b &= -69 \\ a &= 11 \\ c &= -11 \end{aligned}$$

$(a,b,c) = (8, 6, 10)$
 $(a,b,c) = (11, -9, -15)$

(10) (a)

$$A^5 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \Rightarrow A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

(b)

$$A^{-2} = \begin{bmatrix} 9 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow A^{-1} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 1/3 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} \sqrt{3} & 0 & 0 \\ 0 & \sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 1/2 & 0 & 0 \\ 0 & 1/3 & 0 \\ 0 & 0 & 1/6 \end{bmatrix}$$

(11) (a)

$$A = \begin{bmatrix} 3a_{11} & 5a_{12} & 7a_{13} \\ 3a_{21} & 5a_{22} & 7a_{23} \\ 3a_{31} & 5a_{32} & 7a_{33} \end{bmatrix}$$

if D is diagonal then

$$A = BD = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} 3 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 7 \end{bmatrix}$$

$$D = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 7 \end{bmatrix}$$

(12)

$$A^3 = \begin{bmatrix} 1 & 30 \\ 0 & -8 \end{bmatrix} \quad A^2 = \begin{bmatrix} a_{11} & a_{12} \\ 0 & a_{22} \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} \\ 0 & a_{22} \end{bmatrix} = \begin{bmatrix} a_{11}^2 & a_{12}^2 \\ 0 & a_{22}^2 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} a_{11}^3 & a_{12}(a_{11}+a_{22}) \\ 0 & a_{22}^3 \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} \\ 0 & a_{22} \end{bmatrix} = \begin{bmatrix} a_{11}^3 & a_{11}^2 a_{12} + a_{11} a_{12} a_{22} + a_{12}^2 a_{22} \\ 0 & a_{22}^3 \end{bmatrix}$$

(4)

$$a_{11}^3 = 1 \Rightarrow a_{11} = 1$$

$$a_{22}^3 = -8 \Rightarrow a_{22} = -2$$

$$a_{11}^2 a_{12} + a_{11} a_{12} a_{22} + a_{12} a_{22}^2 = 30 \Rightarrow a_{12} - 2a_{12} + 4a_{12} = 30$$

$$3a_{12} = 30 \quad a_{12} = 10$$

$$\therefore A = \begin{bmatrix} 1 & 10 \\ 0 & -2 \end{bmatrix}$$

Q.1.

$$A = \begin{vmatrix} 4 & -1 & 1 & 6 \\ 0 & 0 & -3 & 3 \\ 4 & 1 & 0 & 14 \\ 4 & 1 & 3 & 2 \end{vmatrix}$$

(2) (a) (i) M_{13}

$$= (1) \begin{vmatrix} 0 & 0 & 3 \\ 4 & 1 & 14 \\ 4 & 1 & 2 \end{vmatrix} = (1) (3(0)) = 0$$

$$\boxed{M_{13} = 0}$$

(2) $C_{13} = (-1)^{1+3} M_{13} = 1 \cdot M_{13} = 0$

$$\boxed{C_{13} = 0}$$

(4) (a)

$$A = \begin{vmatrix} 1 & -2 & 3 \\ 6 & 7 & -1 \\ -3 & 1 & 4 \end{vmatrix}$$

$$C_{11} = 29$$

$$C_{12} = 21$$

$$C_{13} = 27$$

$$C_{21} = 11$$

$$C_{22} = 13$$

$$C_{23} = 5$$

$$C_{31} = -19$$

$$C_{32} = 19$$

$$C_{33} = 19$$

$$\boxed{\text{(adj) } A = \begin{vmatrix} 29 & 11 & -19 \\ -21 & 13 & 19 \\ 27 & 5 & 19 \end{vmatrix}}$$

(5)

$$A = \begin{vmatrix} -3 & 0 & 7 \\ 2 & 5 & 1 \\ -1 & 0 & 5 \end{vmatrix} = 5 \begin{vmatrix} -3 & 7 \\ -1 & 5 \end{vmatrix} = 5(-15+7)$$

$$\therefore \boxed{\det(A) = -40}$$

(16)

$$7x_1 - 2x_2 = 3$$

$$3x_1 + x_2 = 5$$

$$A = \begin{vmatrix} 7 & -2 \\ 3 & 1 \end{vmatrix} \quad \det(A) = 13$$

$$A_1 = \begin{vmatrix} 7 & 3 \\ 3 & 5 \end{vmatrix} \quad A_2 = \begin{vmatrix} -2 & 3 \\ 1 & 5 \end{vmatrix}$$

$$\det(A_1) = 26 \quad \& \quad \det(A_2) = -13$$

$$x_1 = \frac{26}{13} = 2 \quad x_2 = \frac{-13}{13} = -1$$

$$A_1 = \begin{vmatrix} 3 & -2 \\ 5 & 1 \end{vmatrix} \quad A_2 = \begin{vmatrix} 7 & 3 \\ 3 & 5 \end{vmatrix}$$

$$\det(A_1) = 13 \quad \det(A_2) = 26$$

$$x_1 = \frac{13}{13} = 1 \quad \& \quad x_2 = \frac{26}{13} = 2$$

$$\boxed{(x_1, x_2) = (1, 2)}$$

(23)

Use Cramer's rule:

$$4x + y + z + w = 6$$

$$3x + 7y - z + w = 1$$

$$7x + 3y - 5z + 8w = -3$$

$$x + y + z + 2w = 3$$

$$A = \begin{vmatrix} 4 & 1 & 1 & 1 \\ 3 & 7 & -1 & 1 \\ 7 & 3 & -5 & 8 \\ 1 & 1 & 1 & 2 \end{vmatrix}$$

$$\det(A) = 4 \begin{vmatrix} 7 & 1 & 1 \\ 3 & -5 & 8 \\ 1 & 1 & 2 \end{vmatrix} - 1 \begin{vmatrix} 3 & 1 & 1 \\ 7 & -5 & 8 \\ 1 & 1 & 2 \end{vmatrix} + 1 \begin{vmatrix} 3 & 7 & 1 \\ 7 & 3 & 8 \\ 1 & 1 & 2 \end{vmatrix} - 1 \begin{vmatrix} 3 & 7 & 1 \\ 7 & 3 & -5 \\ 1 & 1 & 1 \end{vmatrix}$$

$$-1 \begin{vmatrix} 3 & 7 & 1 \\ 7 & 3 & -5 \\ 1 & 1 & 1 \end{vmatrix}$$

(5)

$$\begin{aligned}
 & 4[-126 - 2 + 8] - 1[-54 - 6 + 12] + 1[-6 - 42 + 4] + 1[24 - 84 - 4] \\
 &= 4(-120) - 1(-48) + 1(-44) - 1(-64) \\
 &= -480 + 48 - 44 + 64 \\
 &\therefore \det(A) = -412
 \end{aligned}$$

$$A_4 = \begin{vmatrix} 4 & 6 & 1 & 1 \\ 3 & 1 & 7 & 1 \\ 7 & -3 & -5 & 8 \\ 1 & 3 & 1 & 2 \end{vmatrix}$$

$$= 4 \begin{vmatrix} 1 & 7 & 1 \\ 3 & -5 & 8 \\ 3 & 1 & 2 \end{vmatrix} - 6 \begin{vmatrix} 3 & 7 & 1 \\ 7 & -5 & 8 \\ 1 & 1 & 2 \end{vmatrix} + 1 \begin{vmatrix} 3 & 1 & 1 \\ 7 & -3 & 8 \\ 1 & 3 & 2 \end{vmatrix} + 1 \begin{vmatrix} 3 & 1 & 7 \\ 7 & -3 & -5 \\ 1 & 3 & 1 \end{vmatrix}$$

$$= 4[(-18 + 30 + 12)] - 6[-54 - 6 + 12] + 1[-90 - 6 + 24] + 1[-36 + 12 - 24]$$

$$= 4(24) - 6(-48) + 1(-72) + 1(-72)$$

$$= 96 + 288 - 72 + 72$$

$$\det(A_4) = 384$$

$$\gamma = -\frac{384}{412} = -\frac{192}{206} = -\frac{96}{103}$$

1.6	06
1.7	11
2.1	05