Math 122 Practice Final Fall 2006

Index cards only. No calculators or notes or computers to be used for this part of the exam. This part to be turned in before proceeding with the computer portion.

Find the following derivatives:

1. $f(x) = [\exp(3x+2)+x]^2$

$\frac{df}{dx} = 2\left(\exp(3x+2)+x\right)\left(3\exp(3x+2)+1\right)$

2. $g(x) = \cos(\frac{\sqrt{x^2+1}}{10})$

$\frac{dg}{dx} = -\sin(\frac{\sqrt{x^2+1}}{10})\frac{1}{10}\frac{1}{2}(x^2+1)^{-1/2}2x = \frac{-x\sin(\frac{\sqrt{x^2+1}}{10})}{10\sqrt{x^2+1}}$

Find the following integrals:

3. $\int(\exp(3x+2)+x)(3\exp(3x+2)+1)\,dx = \frac{[\exp(3x+2)+x]^2}{2} + C$

4. $\int_{\ln(2)}^{\ln(3)} \exp(x)\,dx = 1$

Solve the following differential equations:

5. $\frac{dy}{dx} = \frac{x}{y}$; $y(1) = 3.$ $y = \sqrt{8+x^2}$

6. $\frac{dy}{dt} = -\frac{\exp(t)}{y^2}$; $y(0) = 5.$ $y = (128-3e^t)^{1/3}$

7. $\frac{dy}{dt} = -0.003y$; $y(1) = 2.$ $y = \exp(-0.003t+\ln(2)+0.003) = 2e^{-0.003(t-1)}$

Math 122 Practice Final Exam

Full computational aids and open notes portion

1. Find the values of the parameters A, B, omega (ω), and phi (ϕ) that make the fit between the following data

t	f
0.0	18.63
0.1	20.01
0.2	19.18
0.3	16.81
0.4	14.46
0.5	10.12
0.6	8.00
0.7	6.39
0.8	6.27
0.9	8.55
1.0	10.65

and the function

$$f(t) = A + B\sin(\omega t + \phi)$$

as small as possible in the least squares sense.

A = __ 12.9906_

B = _ -6.8119____

ω = __ 4.9671____

ϕ = __ 10.4369____

What are the values of the amplitude, period, and vertical shift for your answer?

The amplitude is 6.8119, the period is 1.265, the vertical shift is 12.99.

2. Find parameters P_0, r and M, that fit the following differential equation

$$\frac{dP}{dt} = -r(P - M)$$

to the data

t	P
0	10.31
1	18.58
2	26.93
3	33.14
4	39.26
5	44.82
6	50.02
7	55.74
8	59.49

P_0 = __10.4697__

r = ___0.090765__

M = __105.554____________

What is the eventual (large time) value of $P(t)$?

105.554

3. Find the area of the region bounded by $y = -3x^4 + 6x^2 + 1$ and $y = x^2$.

5.5217

4. A cat jumps from a ledge that is 2.0 meters off the ground to catch a hummingbird that is hovering x meters above the ground. The highest point of the cat's trajectory is x meters from the ground, i.e. he just manages to catch the bird. He lands on the ground 1.0 second after he begins the jump. What is the value of x and how long after starting the jump does the cat reach the bird?

x = 2.429 meters. The cat reaches this height 0.2959 seconds after jumping.

5. Find the absolute maximum value of the function $y = -3x^4 + 6x^2 + 1$.

$y_{max} = 4$.

6. A certain bacterial population is undergoing Malthusian growth. If its initial mass is 4 grams and 2 hours later it weighs 8 grams, how much does it weigh 7 hours after the start of the experiment?

4*2^(7/2) = 45.25 grams

7. In a certain population, the chances of melanoma for people with zero or very little daily sun exposure is 0.001, and the rate of change of this probability is shown in the table below. John is considering a job that would require 8 hours a day of sun exposure. He currently gets essentially no sun exposure. If he accepts the job, what will his chance of getting melanoma become?

Current sun exposure in hours per day	Increase in Rate of Melanoma Incidence per hour of sun exposure each day
0	0.001
1	0.002
2	0.005
3	0.006
4	0.006
5	0.007
6	0.007
7	0.008
8	0.008

0.043