Math 122 Final Examination Fall 2006

Index cards only. No calculators or notes or computers to be used for this part of the exam. This part to be turned in before proceeding with the computer portion.

Find the following derivatives:

1. $f(x) = \sin^2(3x^2 + x)$

$\frac{df}{dx} =$

2. $g(t) = \exp(-2t + 1)\sin(6t)$

$\frac{dg}{dt} =$

Find the following integrals:

3. $\int_{\sqrt{7}}^{\sqrt{23}} \frac{x}{\sqrt{x^2+2}} dx =$

4. $\int \sin(3x^2 + x)\cos(3x^2 + x)(24x + 4)dx =$

Solve the following differential equations:

5. $\frac{dy}{dx} = 1 - y; \qquad y(0) = 1.$

6. $\frac{dy}{dt} = \exp(t)\exp(y); \qquad y(0) = -\ln(2).$

7. $\frac{dy}{dt} = -\cos(2t); \qquad y(0) = 2.$

NAME_____ Fall 2006
RED ID _____

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Full computational aids and open notes portion

For problems 1 and 2, go to the website for this course and click on the link that gives the data for the final. Use the data you get to do the following problems

1. Find the values of the parameters A, B, omega (ω), and phi (ϕ) that make the fit between the given data and the function

$$f(t) = A + B\sin(\omega t + \phi)$$

as small as possible in the least squares sense.

A = _____ B = _____ ω = _____ ϕ = _____

What are the values of the amplitude, period, and vertical shift for your answer?

2. Find values of the constant P_0, k and M, that fit the following differential equation

$$\frac{dP}{dt} = k\,(1 - P/M)$$

to the given data by minimizing the sum of the squares of the errors.

P_0 = _____ k = _____ M = _____

What is the eventual (large time) value of $P(t)$?

3. Find the absolute minimum and maximum values of the function $y = \exp(\sin(x^2))$.

NAME_____ Fall 2006
RED ID _____

4. Find the area of the region bounded by $y = \exp(\sin(x^2))$ and $y = x^2$.

5. A cat jumps from a ledge to catch a hummingbird that is hovering 3 meters above the ground. The highest point of the cat's trajectory is 3 meters from the ground, i.e. he just manages to catch the bird. He catches the bird 0.25 seconds after starting the jump. How high was the ledge and how long does he spend in the air?

6. A certain bacterial population is undergoing Malthusian growth. If its initial mass is 6 grams and 5 hours later it weighs 8 grams, how much did it weigh 3 hours after the start of the experiment?

7. The concentration of bacteria X is measured at several points along a certain sewer pipe. The pipe contains a uniform liquid depth that amounts to 1 liter per foot of pipe. Assuming that in each 10 ft stretch the concentration is constant at the initial value, find the total number of bacteria in the first 50 feet of pipe.

Distance in feet from the sewer opening	Concentration of Bact X (Number of Bacteria per ml)
0	$3*10^6$ bact/ml
10	$4*10^6$ bact/ml
20	$7*10^6$ bact/ml
30	$2*10^6$ bact/ml
40	$1*10^6$ bact/ml
50	$2*10^6$ bact/ml
60	$4*10^6$ bact/ml
70	$5*10^6$ bact/ml
80	$7*10^6$ bact/ml