Math 336 33 pts/problem

Practice Test 3

1. Given that a population moves between two classes Rich and Poor according to the transition matrix

$$\begin{bmatrix} 0.2 & 0.1 \\ 0.8 & 0.9 \end{bmatrix}$$

i.e. if a family is rich in one generations it has a 20% chance of being rich at the next generation, etc..

a. Write the recursion relation for the population vector $P\,(t+1) = \begin{bmatrix} p_{rich}(t+1) \\ p_{poor}(t+1) \end{bmatrix}$ one time step from now, given the current population vector $P\,(t) = \begin{bmatrix} p_{rich}(t) \\ p_{poor}(t) \end{bmatrix}$.

b. Find the stationary distribution for this Markov Chain.

c. Find the other eigenvector of the matrix and normalize it so the component corresponding to the rich class (the first component) equals 1. Also find the corresponding eigenvalue.

d. If everyone in the population starts out rich, i.e. the population starts at the distribution $P_0 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, write this initial distribution as a linear combination of the two eigenvectors you found in a and b.

e. Use your answer to c to write down the distribution after 100 time steps in terms of the two eigenvectors.

2. Consider $x(n+2) = 12\,x(n+1) - 35\,x(n)$.
   a. If $x(0)$=2 and $x(1)$=12, find a closed form formula for $x(n)$.
   b. Define the vector $y(n) = \begin{bmatrix} x(n) \\ x(n+1) \end{bmatrix}$ Show that y satisfies the recursion relation
   $$y(n+1) = \begin{bmatrix} 0 & 1 \\ -35 & 12 \end{bmatrix} y(n).$$
   c. Compare the eigenvalues of the matrix above to the characteristic values of the original second order equation.

3. Consider a population made up of three age cohorts that evolve according to the Leslie matrix
$$\begin{bmatrix} 0 & 0.8 & 3.5 \\ 0.8 & 0 & 0 \\ 0 & 0.9 & 0 \end{bmatrix}.$$
   a. What fraction of the population makes it from the second age cohort to the third age cohort in one time step?
   b. Compute the normalized dominant eigenvector of this matrix. (Normalized means rescaled so non-negative entries add to 1.)
   c. If the initial population is $\begin{bmatrix} 100 \\ 0 \\ 0 \end{bmatrix}$, find the normalized population after 100 steps.