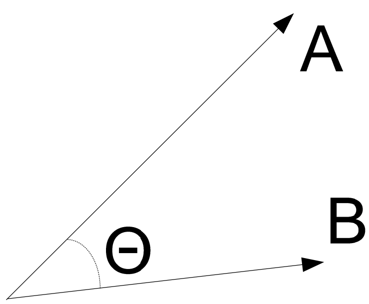
Problem Set 1

Introduction to Matlab and review of several mathematical concepts
Due date is posted on the online course calendar

Work through chapter 2 of the introduction to Matlab. *After doing so*, use your newfound knowledge to answer the following questions. Use the diary feature of Matlab to keep a log of your work. Edit the diary to remove failed attempts and turn in a log showing your work. Circle the answer to each problem.

In some cases, you will be asked to use material that is not covered in the tutorial. When you see the text "(doc *topic*)," it means that typing `doc topic` at the Matlab prompt will bring up the appropriate documentation in the help browser. For example, to learn about the diary feature, type `doc diary`.

1. (25 points) The dot product of two vectors is related to the cosine of the angle between them: $A \cdot B = |A||B| cos(\theta)$. Assuming that vector A = [3 4]' and B = [9 2]' [1], create variables for A and B. Find the angle (in radians) between the two vectors. Helpful built in functions are norm, acos, and dot. Use the help browser to read about these functions.



2. (15 points) Given matrix *A* and vector *b*, the equation *Ax*=*b* can be solved for *x* by methods such as Gaussian elimination. Matlab solves such problems using the matrix division operator \. When the columns of *A* represent a basis set and *b* is a vector in $\Re^N$, then we can learn the coefficients to each basis in the new basis *A* by solving for *x*.

   As an example, consider the trivial basis

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

[1] We will use ' or ^{T} to denote transpose throughout this course

which is our normal 3 dimensional basis set with the roles of the x, y, and z axes permuted. The vector [3 4 5]' becomes [5 3 4]', and are of course coefficients for the new basis set:

$$5\begin{bmatrix}0\\0\\1\end{bmatrix}+3\begin{bmatrix}1\\0\\0\end{bmatrix}+4\begin{bmatrix}0\\1\\0\end{bmatrix}$$

Use Matlab to show that the coefficients in the new basis set are indeed [5 3 4]'.

3. (25 points) You should recall that some matrices have characteristic vectors, or eigenvectors. Eigenvectors obey the following property: $Ae = \lambda e$ which means that *e* is an eigenvector of A if the multiplication results in some scalar multiple λ (called the eigenvalue) of *e*. These eigenvectors and their associated eigenvalues can be used to understand how the matrix multiplication Ax maps x to its image (output).

   For the matrices that we will consider in this class, we should always be able to find a set of eigenvectors that form a basis set for the input space. We can thus determine how much of x lies in the direction of each eigenvector using the techniques we saw in the previous problem.

   Suppose that a 3x3 matrix has the eigenvectors e_1, e_2, and e_3, and the eigenvalues λ_1, λ_2, and λ_3. We can use the eigenvectors in the same role as the basis and determine the coefficients to our eigenvectors needed to represent *x*. Thus $x = \hat{x}(1)e_1 + \hat{x}(2)e_2 + \hat{x}(3)e_3$.

   It turns out that *Ax* is simply $Ax = \hat{x}(1)\lambda_1 e_1 + \hat{x}(2)\lambda_2 e_2 + \hat{x}(3)\lambda_3 e_3$. If we wish to raise A to a power, then $A^n x = \hat{x}(1)\lambda_1^n e_1 + \hat{x}(2)\lambda_2^n e_2 + \hat{x}(3)\lambda_3^n e_3$.

   Given the following matrix:

$$A=\begin{bmatrix}0.0806 & -0.0053 & -0.0003\\ -0.0053 & 0.0827 & -0.0029\\ -0.0003 & -0.0029 & 0.0833\end{bmatrix}$$

   and the vector *x*=[.8 .4 .2]' do the folloing:

   a. plot a line from the origin to *Ax* in three dimensions. (HINT: 3D plotting is similar to the 2D plot covered in the material. See plot3 in the Matlab help browser)
   b. Determine the eigenvalues and eigenvectors using Matlab's builtin eig function (see section 4.1 of the tutorial).

In the next set of steps, you will show graphically that $Ax = \hat{x}(1)\lambda_1 e_1 + \hat{x}(2)\lambda_2 e_2 + \hat{x}(3)\lambda_3 e_3$.

c. Compute $\hat{x}$, then compute each $\hat{x}(i)\lambda_i e_i$. Draw a dotted line (doc linespec) from the the origin to $\hat{x}(1)\lambda_1 e_1$. Continue the line from $\hat{x}(1)\lambda_1 e_1$ to $\hat{x}(1)\lambda_1 e_1 + \hat{x}(2)\lambda_2 e_2$. Draw a final segment from $\hat{x}(1)\lambda_1 e_1 + \hat{x}(2)\lambda_2 e_2$ to $\hat{x}(1)\lambda_1 e_1 + \hat{x}(2)\lambda_2 e_2 + \hat{x}(3)\lambda_3 e_3$. To prevent Matlab from erasing the figure each time you plot a new line, use the command "hold on" once after plotting the original line from the origin to *Ax*. If you ever need to clear a held figure, you can type "hold off" and the next plot will overwrite it, or simply "clf" (clear figure).

In addition to your code to create this, you should turn in a copy of your figure. If you are using Windows, you can copy and paste into Word. On UNIX/linux, you can use the File/Save As menu to save the figure as a graphics file. Many types of files are supported; I would suggest a common one such as JPEG. You may find it helpful to use a loop construct for this (section 3.1), but you are not required to do so.

4. (35 points) The covariance of two variables is defined as:

$$\text{cov}(x, y) = E\left[(X - E[X])(Y - E[Y])'\right]$$

where *E* denotes the expected value of a variable. When it is not possible to compute the expected value of the variables, the covariance is frequently estimated as:

$$\widehat{cov}(X,Y) = \frac{1}{N-1}\sum_{i=1}^{N}(x_i - \mu_x)(y_i - \mu_y)$$

where μ_x and μ_y denote the expected values (mean or average) of X and Y respectively. When data consists of multiple variables $X_1, X_2, \ldots X_k$, it is common to build a matrix with the covariances:

$$\begin{bmatrix} \text{cov}(X_1, X_1) & \text{cov}(X_1, X_2) & \cdots & \text{cov}(X_1, X_k) \\ \text{cov}(X_2, X_1) & \text{cov}(X_2, X_2) & \cdots & \text{cov}(X_2, X_k) \\ \cdots & & \ddots & \vdots \\ \text{cov}(X_k, X_1) & \text{cov}(X_k, X_2) & \cdots & \text{cov}(X_k, X_k) \end{bmatrix}$$

a. (15 points) Prove that the covariance matrix is always symmetric.
b. (5 points) Look up the definition of variance if you do not remember it. Prove that the diagonal elements are the variance. (Note, we are using the unbiased estimate which weights by $1/_{N-1}$, biased versions weight by $1/_N$.)
c. (15 points) The function mvnrnd can take random draws from a multivariate normal distribution. Use doc mvrnd to see an example of how to use this function which is part of the Statistics Toolbox. If you

have a student version of Matlab, this will not be included in your distribution and you will need to use one of the university's copies (e.g rohan or a CS lab). Using a mean vector of [0 0]', and covariance matrices of your choice (do not forget that they must be symmetric), generate and plot enough distributions that you understand:

1. What types of distributions result when the off diagonals are zero.
2. What types of distributions result when the off diagonals are positive.
3. What types of distributions result when the off diagonals are negative.

In addition to your descriptions, include a sample plot of a draw from each type.