Be able to use the following terminology

- eigenvalue, eigenvector (be able to define these also).
- basis for eigenspace.
- characteristic polynomial, characteristic equation.
- similar matrices.

Eigenvectors and Diagonalization

- Let \( A \) be an \( n \times n \) matrix. You should be able to do the following.
  - Compute the characteristic polynomial of \( A \).
  - Find the eigenvalues of \( A \), when the characteristic polynomial is easily factored.
  - Find a basis for the eigenspace for each eigenvector.
  - Diagonalize \( A \) given \( n \) linearly independent eigenvectors.
  - When \( A \) is \( 2 \times 2 \), and has complex eigenvalues, find a rotation-scaling matrix that is similar to \( A \). That is, if \( a \pm bi \) are the eigenvalues, find \( P \) such that \( P^{-1}AP = \begin{bmatrix} a & -b \\ b & a \end{bmatrix} \).
- Be able to use and understand the meaning of the main theorems.
  - \( A \) is diagonalizable if and only if it has \( n \) linearly independent eigenvectors.
  - If \( A \) has \( n \) distinct eigenvalues it is diagonalizable.
  - A matrix \( A \) is invertible if and only if 0 is not an eigenvalue of \( A \).
  - Similar matrices have the same characteristic polynomial, and therefore the same eigenvalues with the same multiplicities.
- Be able to apply eigenvector analysis to a dynamical system.
  - Be able to classify a \( 2 \times 2 \) matrix \( A \):
    - Is the origin an attractor, a repellor, or a saddle point? Is \( A \) a rotation-contraction or a rotation-dilation? The latter cases occur when the eigenvalues are not real.
  - Be able to identify the long term behavior of a dynamical system, given the eigenvalues and eigenvectors.
  - Be able to write a transition matrix for a dynamical system given information about population changes.