Solving Linear Systems. Know how to:

- Transform a system of linear equations into a matrix equation.
- Transform a traffic flow problem into a linear system, and then into a matrix equation.
- Solve a system using Gaussian elimination.
- Explain the steps that you use (row replacement steps and row exchange steps).
- Write a vector \( x \) as a sum of vectors \( v_1, \ldots, v_m \) (or check that it can’t be done) using Gaussian elimination.
- Check whether vectors \( v_1, \ldots, v_n \) are linearly independent using Gaussian elimination.
- Invert a matrix using Gaussian elimination.

The Language of Linear Transformations. Let \( T : \mathbb{R}^n \rightarrow \mathbb{R}^m \) be the linear transformation given by the \( m \times n \) matrix \( A \). Let \( b \in \mathbb{R}^n \).

- Be able to translate transformations on \( \mathbb{R}^2 \), stated as rotations, reflections, or dilations/contractions into matrix form. Be able to work with diagrams of the image of a transformation \( \mathbb{R}^2 \rightarrow \mathbb{R}^2 \) (§1.9#1-14).
- The set of \( x \) satisfying \( T(x) = b \) is the same set as the set of \( x \) satisfying \( Ax = b \). be able to translate back and forth between the two: (§1.9#17-28).
- **Theorem 6:** Let \( p \) be one solution to \( Ax = b \). The solution set of \( Ax = b \) is all \( p + v_h \) where \( v_h \) varies among the solutions to \( Ax = 0 \).
- **Theorem 11:** \( T \) is a one-to one function if and only if the only solution to \( Ax = 0 \) is the zero vector in \( \mathbb{R}^n \). [This follows directly from Theorem 6. When there is only one homogeneous solution, there is at most one solution to \( Ax = b \).]
- There are many ways to express the same essential idea. In the following table, as you read across each row you see different ways of saying a particular concept. The final line contains most of the Invertible Matrix Theorem (Theorem 8 in Chapter 2).
<table>
<thead>
<tr>
<th>$Ax = b$ has a solution for all $b \in \mathbb{R}^m$</th>
<th>RREF of $A$ has a pivot in each row</th>
<th>The columns of $A$ span $\mathbb{R}^m$</th>
<th>The linear transformation $x \mapsto Ax$ is onto</th>
<th>There is an $m \times m$ matrix $Z$ such that $ZA = I_m$</th>
<th>Note: $n \geq m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Ax = 0$ has one solution, $x = 0$</td>
<td>RREF of $A$ has a pivot in each column</td>
<td>The columns of $A$ are linearly independent in $\mathbb{R}^m$</td>
<td>The linear transformation $x \mapsto Ax$ is one-to-one</td>
<td>There is an $n \times n$ matrix $Z$ such that $AZ = I_n$</td>
<td>Note: $n \leq m$</td>
</tr>
<tr>
<td>$Ax = b$ has exactly one solution for each $b \in \mathbb{R}^m$</td>
<td>RREF of $A$ has a pivot in each row and each column</td>
<td>The columns of $A$ are linearly independent and span $\mathbb{R}^m$</td>
<td>The linear transformation $x \mapsto Ax$ is one-to-one and it onto</td>
<td>There is a $n \times n$ matrix $Z$ such that $AZ = ZA = I_n$</td>
<td>Note: $n = m$</td>
</tr>
</tbody>
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