Suggestions for Preparing for the Final Exam

I. Understand fundamental logic:
   - Truth tables for $\land, \lor, \implies$, xor.
   - Be able to show logical equivalence using a truth table.
   - Know some important equivalences
     (Thm. 1.1.1 and $p \implies q \equiv \neg p \lor q$ and $p \implies (q \lor r) \equiv (p \land \neg q) \implies r$).
   - Know how to negate a statement (Important!).
   - Pay particular attention to quantifiers, existential ($\exists$) and universal ($\forall$).

II. Know the basics of set theory!
   - Definitions of subset, intersection, union, set difference, complement.
   - Definitions of power set, Cartesian product, partition.
   - Be able to prove one set is a subset of another.

III. Know the basics of number theory.
   - The definition of divides and divisibility properties.
   - Be able to state and use the Quotient-Remainder Theorem.
   - Be able to state and use the Unique Factorization Theorem.
   - Know the classic proofs by contradiction
     - $\sqrt{p}$ is irrational for $p$ prime.
     - The sum of a rational number and an irrational number is irrational.

IV. Functions and relations.
   - Know the definitions: Relation, inverse of a relation, function.
     For functions: injective (one-to-one), surjective (onto) and bijective.
   - Be able to determine when a relation is a function, and if so, when it is injective, surjective, or bijective.
   - Be able to work with lists, tables, arrow diagrams and formulas to define relations.
   - Be able to find the inverse of a bijective function (e.g. $f(x) = 2x + 7$).
   - Give examples of functions satisfying various properties
     (7.3 #4-10 (2nd Ed.) 7.2 #5-14 (3rd Ed.)).
V. Relations on a set.

- Know how to use a table, a list of elements or an arrow diagram (also called a directed graph) to represent a relation on $X$. (The arrow diagram is different from that for a relation from $X$ to $Y$).
- Verify or prove that a relation $R$ is reflexive (ditto for symmetric, transitive, antisymmetric, an equivalence relation, or a partial order).
- For a relation $R$ on $A$, be able to find the smallest relation containing $R$ which is symmetric (ditto for reflexive, transitive, an equivalence relation, a partial order).
- Know the standard examples of equivalence relations (mod $n$, 10.3.10 and exercises 10.3 #18, 19, 22, 23 3rd Ed., 10.3 #15, 16, 19, 20 2nd Ed.).
- Know the standard examples of partially ordered sets: $\leq$ for the integers (or rationals) divides on the integers; $\mathcal{P}(A)$ for a set $A$; $D_n$; (10.5 #16, 17, 18, 19, 20, 21 both Eds.).
- Draw Hasse diagrams for a poset. Find minimal and maximal elements of a poset.

VI. Know the basics of recursion and induction!

- State the well-ordering principle.
- State the principle of induction.
- Be able to use summation and product notation.
- Find the first several terms of a sequence given the initial terms and the recurrence formula.
- Know the formulas for the following sums:
  - The sum of a geometric sequence.
  - The sum of the first $n$ integers.

VII. Know how to prove by standard induction! (I won’t test strong induction.)

- Use full sentences.
- State the predicate.
- Prove the base step.
- State the assumption for the inductive step.
- Do the inductive step.

VIII. Know how to count!

- Know the inclusion-exclusion formula and be able to apply it and use a Venn diagram to illustrate.
- Know the 4 ways to choose and the formulas for 3 of them (I won’t test ”order unimportant, repetition allowed”).
- Poker hands (I will describe the hand, and I may give you a strange deck or strange hand).