Suggestions for preparing for the Second Exam

I. Things you should know about the integers and rational numbers:

- Be able to use (and recognize that you are using) commutativity, associativity, the additive and multiplicative identity, the additive inverse (and, for the rationals, the multiplicative inverse), distributivity.
- Properties of $<$. For example $a < b$ implies $a + c < b + c$.
- Be able to define prime, composite, divides, floor, ceiling.
- State the well-ordering principle.
- State the principle of induction.

II. Know the statements of the following theorems and know how to apply them (as in webworks problems):

- Quotient-remainder theorem.
- The unique factorization theorem.

III. Know these classic proofs.

- Transitivity of divides.
- If $a$ divides $b$ and $a$ divides $c$ then $a$ divides $b + c$.
- There exist an infinite number of primes (by contradiction).
- $\sqrt{2}$ is irrational (by contradiction).
- The sum of a rational number and an irrational number is irrational (by contradiction).

IV. Sequences and recursion.

- Be able to use summation and product notation.
- Be able to use recursive formulas.
- Find the first several terms of a sequence given the initial terms and the recurrence formula.
- Find the formula for the $n$th term as a function of $n$ for some simple examples.

V. Know the formulas for the following sums:

- The sum of a geometric sequence.
- The sum of the first $n$ integers.
VI. Know how to prove by induction!

- Use full sentences.
- State the predicate.
- Prove the base step.
- State the assumption for the inductive step.
- Do the inductive step.