Suggestions for Preparing for the Final Exam

I. Understand fundamental logic:
   • Truth tables for $\land$, $\lor$, $\implies$, xor.
   • Be able to show logical equivalence using a truth table.
   • Know some important equivalences
     (Thm. 1.1.1 and $p \implies q \equiv \neg p \lor q$ and $p \implies (q \lor r) \equiv (p \land \neg q) \implies r$).
   • Know how to negate a statement (Important!).
   • Pay particular attention to quantifiers, existential ($\exists$) and universal ($\forall$).

II. Know the basics of set theory!
   • Definitions of subset, intersection, union, set difference, complement.
   • Definitions of power set, Cartesian product, partition.
   • Be able to prove one set is a subset of another.

III. Know the basics of number theory.
   • The definition of divides and divisibility properties.
   • Be able to state and use the Quotient-Remainder Theorem.
   • Be able to state and use the Unique Factorization Theorem.
   • Know the classic proofs by contradiction
     – There exist an infinite number of primes.
     – $\sqrt{p}$ is irrational for $p$ prime.
     – The sum of a rational number and an irrational number is irrational.

IV. Functions and relations.
   • Know the definitions: Relation, inverse of a relation, function. For functions: injective (one-to-one), surjective (onto) and bijective.
   • Be able to determine when a relation is a function, and if so, when it is injective, surjective, or bijective.
   • Be able to work with lists, tables, arrow diagrams and formulas to define relations.
   • Be able to find the inverse of a bijective function (e.g. $f(x) = 2x + 7$).
• Give examples of functions satisfying various properties
  (7.3 #4-10 (2nd Ed.) 7.2 #5-14 (3rd Ed.)).

V. Relations on a set.

• Know how to use a table, a list of elements or an arrow diagram (also called a directed graph) to represent a relation on \( X \). (The arrow diagram is different from that for a relation from \( X \) to \( Y \)).

• Verify or prove that a relation \( R \) is reflexive (ditto for symmetric, transitive, antisymmetric, an equivalence relation, or a partial order).

• For a relation \( R \) on \( A \), be able to find the smallest relation containing \( R \) which is symmetric (ditto for reflexive, transitive, an equivalence relation, a partial order).

• Know the standard examples of equivalence relations
  (mod \( n \), 10.3.10 and exercises 10.3 #18, 19, 22, 23 3rd Ed., 10.3 #15, 16, 19, 20 2nd Ed.).

• Know the standard examples of partially ordered sets: \( \leq \) for the integers (or rationals) divides on the integers; \( \mathcal{P}(A) \) for a set \( A \); \( D_n \); (10.5 #16, 17, 18, 19, 20, 21 both Eds.).

• Draw Hasse diagrams for a poset. Find minimal and maximal elements of a poset.

VI. Know the basics of recursion and induction!

• State the well-ordering principle.

• State the principle of induction.

• Be able to use summation and product notation.

• Find the first several terms of a sequence given the initial terms and the recurrence formula.

• Know the formulas for the following sums:
  – The sum of a geometric sequence.
  – The sum of the first \( n \) integers.

VII. Know how to prove by induction!

• Use full sentences.

• State the predicate.

• Prove the base step.

• State the assumption for the inductive step.

• Do the inductive step.

VIII. Know how to count!

• Know the 4 ways to choose and the formulas for 3 of them (I won’t test ”order unimportant, repetition allowed”).

• State the binomial theorem, and use it to find a particular coefficient in a binomial expansion (6.7 #4, 8).

• Poker hands (I will describe the hand, and I may give you a strange deck).