• Be able to precisely define the following terms. Be careful about the logic in the definition!
  – Group, subgroup, cyclic group, generators of a group.
  – Order of an element, order of a group.
  – Homomorphism, isomorphisms, automorphism, inner automorphism.
  – Center of a group, centralizer of an element.
  – Normal subgroup.

• Here are the key theorems; be able to use them.
  – Theorem 7.8: the order of an element.
  – Theorem 7.10: properties determining a subgroup.
  – Theorems 7.18, 7.28: cyclic groups.
  – Theorem 7.23, 7.24: Properties of cosets (see my in class version).
  – Theorems 7.26, 7.27: (Lagrange) the order of subgroups of a finite group.
  – Theorems 7.34, 7.36: Properties of normal subgroups and quotients.
  – 7.41, 7.42, 7.43, 7.44: Homomorphism and isomorphism theorems.
  – Let \( \phi : G \rightarrow H \) be a homomorphism. You should be able to prove these.
    * If \( B \) is a subgroup of \( B \) then \( \phi^{-1}(B) \) is a subgroup of \( G \). In addition, if \( B \) is normal
      then \( \phi^{-1}(B) \) is normal.
    * If \( A \) is a subgroup of \( G \) then \( \phi(A) \) is a subgroup of \( H \). (Caution: If \( A \) is normal,
      \( \phi(A) \) may not be!)
  – You should be able to prove some of the simpler results about abelian groups and the
    order of an element (Sec. 7.2).

• Known how to work with the standard examples.
  – \((\mathbb{Z}_n, +), (U_n, \cdot)\).
  – \(\text{Gl}(2, \mathbb{Q}), \text{Sl}(2, \mathbb{Q})\) and the matrix groups over \(\mathbb{Z}_p\) for \(p\) prime.
  – The symmetric group \(S_n\), the dihedral group \(D_n\).
  – Subgroups of the above, such as the quaternions.
  – Review the classification of groups of order < 8 that we did in class.