Abstract Algebra B
Math 521B
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Review for third exam

• Be able to define precisely the following terms. Be careful about the logic in the definition!
  – transcendental element, algebraic element, minimal polynomial (Thm. 10.6).
  – algebraic extension, simple extension, finitely generated extension, splitting field.

• Know some standard examples over \( \mathbb{Q} \) and \( \mathbb{R} \).
  – Find the minimal polynomial of some simple examples (e.g. \( \sqrt{2 + i} \)).
  – Find bases for extensions: (e.g. \( \mathbb{Q}(\sqrt{2}, \sqrt{3}) \) over \( \mathbb{Q} \)).
  – Know the theorems and exercises about the dimension about a composite extension
    (Thm 10.10, Ex. 10.#7, 8, 9, 11, 13).

• Know how to work with finite fields.
  – Know the Freshman’s rule (see also Ex. 10.6# 10, 12).
  – Know the existence and uniqueness theorem.
  – Know that the multiplicative group of a finite field is cyclic.
  – Know that a finite field is a simple extension of a prime field.
  – Given an irreducible polynomial whose root \( \alpha \) generates the multiplicative group of \( \mathbb{F}_{p^n} \),
    construct the dictionary between powers of \( \alpha \) and polynomials in \( \alpha \).
  – Use the dictionary to compute.