Rings

• Know the definitions:
  – ring, commutative, identity, field;
  – unit, zero divisor, characteristic;
  – homomorphism, isomorphism.

• Know how to:
  – Prove that a subset of a ring is a subring, (or show that it isn’t).
  – Prove that a function is a homomorphism, or isomorphism (or show that it isn’t).
  – Show that two rings can’t be isomorphic, because they have some difference in structure.
  – Identify the units and zero divisors in a ring.

• Know how to construct new rings from old and to compute in these rings.
  – The Cartesian product of rings $R$ and $S$ is a ring $R \times S$.
  – The $2 \times 2$ matrices over a ring $R$ form a ring, which we write $M(R)$.
  – We also have the polynomial ring, $R[x]$ over a ring $R$.

Polynomial rings over a field $F$

• Know the special properties of $F[x]$, and that is is similar to $\mathbb{Z}$.
  – Division theorem.
  – Euclidean algorithm.
  – Prime iff irreducible.
  – Unique factorization.

• Know the relationship between roots and factors.
  – In $F[x]$, the remainder when $f(x)$ is divided by $(x - a)$ is $f(a)$.
    So $x - a$ is a factor of $f(x)$ iff $a$ is a root of $f(x)$.
  – Know how to test whether a polynomial in $\mathbb{Z}_p[x]$ of degree 2 or 3 is irreducible.