Linear Algebra
Math 254
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Review for third exam
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Be able to use the following terminology

- eigenvalue, eigenvector (be able to define these also).
- basis for eigenspace.
- characteristic polynomial, characteristic equation.
- similar matrices.

Eigenvectors and Diagonalization

- Let A be an $n \times n$ matrix. You should be able to do the following.
  - Compute the characteristic polynomial of A.
  - Find the eigenvalues of A, when the characteristic polynomial is easily factored.
  - Find a basis for the eigenspace for each eigenvector.
  - Diagonalize A given n linearly independent eigenvectors.
  - When A is 2×2, and has complex eigenvalues, find a rotation-scaling matrix that is similar to A. That is, if $a \pm bi$ are the eigenvalues, find P such that $P^{-1}AP = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$.
- Be able to use and understand the meaning of the main theorems.
  - A is diagonalizable if and only if it has n linearly independent eigenvectors.
  - If A has n distinct eigenvalues it is diagonalizable.
  - A matrix A is invertible if and only if 0 is *not* an eigenvalue of A.
  - Similar matrices have the same characteristic polynomial, and therefore the same eigenvalues with the same multiplicities.
- Be able to apply eigenvector analysis to a dynamical system.
  - Be able to classify a 2×2 matrix A:
    Is the origin an attractor, a repellor, or a saddle point? Is A a rotation-contraction or a rotation-dilation? The latter cases occur when the eigenvalues are not real.
  - Be able to identify the long term behavior of a dynamical system, given the eigenvalues and eigenvectors.
  - Be able to write a transition matrix for a dynamical system given information about population changes.