

Information Theory

Problem Set 3

- 1) Consider optimal sequences l_1, \dots, l_m base n .
 1. For $m = 11$ and $n = 2$ find all optimal sequences.
 2. For $m = 11$ and $n = 3$ find all optimal sequences.
 3. Write Maple code to compute all optimal sequences for given m, n .
 4. For $n = 2$, use Maple to compute the number of optimal sequences for given m as m varies. See if you can guess the growth rate of this function.

- 2) Consider the source $S = \{A, B, \dots, H\}$ with frequencies, $f_A = .4, f_B = .2, f_C = .15, f_D = .1, f_E = .05, f_F = .04, f_G = .03, f_H = .03$.
 1. Find a minimal variance Huffman code, a Shannon code and a Shannon-Fano-Elias code for the binary alphabet.
 2. Compute the expected lengths for these codes and compare with the entropy.
 3. What would the expected length be for the Shannon code on S^2 . You'll probably want to use MAPLE to compute it.
 4. Find a minimal variance Huffman code for the tertiary alphabet.

- 3) (Sayood 3.8.6) In many communication channels it is desirable that the number of 0's and 1's transmitted over the channel are about the same. Do the codes that you designed in problem 1 above satisfy this desire?

- 4) (Cover 5 ex. 10) Codes meeting the entropy bound.
 1. Suppose a tertiary code on a source S satisfies: $\text{Exp.length} = H_3(S)$. Show that $\#|S|$ is odd and each f_i is an integer power of 3.
 2. Give examples for tertiary codes with $m = 7, 8$ meeting the entropy bound and not meeting the bound.
 3. Prove: An n -ary code on a source of m symbols meets the entropy bound $H_n(S)$ if and only if each frequency is an integral power of n .

- 5) (Cover 5 ex. 9) Optimal codes far from the entropy bound. Give an example of source frequencies for which the expected length of an optimal code is close to $H(S) + 1$. For any ϵ , construct a source S for which the optimal code length is at least $H(S) + 1 - \epsilon$.

- 6) Consider a source S with m elements and frequencies f_i for $i = 0, \dots, m - 1$. For any positive integer N , we consider $S^N \subset [0, 1)$. Arithmetic coding associates an interval to each $\sigma \in S^N$. Write a MAPLE program that graphs that interval as a function of the frequency list (or the cumulative frequency list F) and N . Produce graphs for
 1. $f_0 = 3/4, f_1 = 1/8, f_2 = 1/8$ and $N = 1, 2, 4, 8$.
 2. $f_0 = 7/8, f_1 = 7/64, f_2 = 1/64$ and $N = 1, 2, 4, 8$.

- 7) Write a MAPLE program implementing some form of arithmetic encoding and decoding in binary.