

# DISCRETE MATHEMATICS

## Math 245

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### Practice Exam

Do all problems. Show your work!

I. Logical arguments are based on a logic statement being a tautology. State the tautology that justifies the argument:

$$\begin{array}{c} p \\ q \\ \hline (p \wedge q) \implies r \\ r \end{array}$$

Show that it is a tautology using a truth table.

Give an example using statements in mathematical or everyday language illustrating the use of this equivalence.

II. For the following statement in English:

Everyone at the party was beautiful or smart.

- Negate the statement. (Don't just write "It is not true that ...")
- Translate the original statement into formal logic.
- Negate the formal statement and simplify. (That is, don't just write " $\sim (\dots)$ ."

III. Are the following equivalent:  $\exists x \in D : P(x) \wedge Q(x)$ , versus  $(\exists x \in D : P(x)) \wedge (\exists x \in D : Q(x))$

Create similar problems to this one by varying the existential and the logic operator ( $\wedge$ ).

IV State the quotient-remainder theorem. State the unique factorization theorem. State the theorem about a prime dividing a product.

V Define *divides*, *composite*, and *floor*.

VI. Consider the following statement:

For any integer  $m$  and any rational number  $r$ ,

$$\lceil m - r \rceil = m - \lfloor r \rfloor$$

- Illustrate with an example.
- Show that the statement is not true if  $m$  is allowed to be any rational number.
- Prove the statement. Use the definitions of floor and ceiling!