

DISCRETE MATHEMATICS

Math 245

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Suggestions for Preparing for the Final Exam

I. Understand fundamental logic.

- Truth tables for \wedge , \vee , \implies , xor.
- Variety of ways to express conditionals in English.
- Variety of ways to express universal and existential statements.
- Know how to negate a statement (Important!).

II. Know the statements of the following theorems:

- Quotient-remainder theorem.
- The unique factorization theorem.

III. Know the classic proofs by contradiction:

- There exist an infinite number of primes.
- $\sqrt{2}$ is irrational.
- The sum of a rational number and an irrational number is irrational.

IV. Know the basics of set theory!

- Subset. Intersection, union, set difference, complement.
- Power set, Cartesian product, partition.
- Relation, inverse of a relation, function. Injective (one-to-one), surjective (onto) and bijective functions.
- Reflexive, symmetric, antisymmetric, asymmetric, transitive.
- Equivalence relation, equivalence class.
- Partial order (poset). For posets, comparable, chain, total order, maximal, minimal, least, greatest.

V. Know your relations.

- Know how to use tables, graphs and lists of elements to represent a relation.
- Verify or prove that a relation R is reflexive (ditto for reflexive, transitive, equivalence relation, partial order).

- For a relation R on A , be able to find the smallest relation containing R which is symmetric (ditto for reflexive, transitive, an equivalence relation, a partial order).
- Know the standard examples of equivalence relations ($\text{mod } n$, 10.3 #19, 20, 22).
- Know the standard examples of partially ordered sets: \leq for the real numbers; divides on the integers; $\mathcal{P}(A)$ for a set A ; D_n ; (10.5 #18, 19, 20).
- Draw Hasse diagrams for a poset. Find minimal and maximal elements of a poset.

VI. Functions as relations.

- Determine when a relation is a function, and if so, when it is injective, surjective, or bijective.
- Find the inverse relation of a function. Is it a function, injective, surjective?
- Give examples of functions satisfying various properties (7.3 #4-10).

VII. Know the basics of recursion and induction!

- State the well-ordering principle.
- State the principle of induction.
- Be able to use summation and product notation.
- Find the first several terms of a sequence given the initial terms and the recurrence formula.

VIII. Know the formulas for the following sums:

- The sum of a geometric sequence.
- The sum of the first n integers.

IX. Know how to prove by induction!

- Use full sentences.
- State the predicate.
- Prove the base step.
- State the assumption for the inductive step.
- Given the initial terms and the recurrence formula for a sequence and given a formula for the n th term as a function of n prove that the latter is correct.

X. Know how to count!

- Poker hands (I will describe the hand, and I may give you a strange deck).
- State the binomial theorem, and use it to find a particular coefficient in a binomial expansion (6.7 #4, 8).
- Know the 4 ways to choose and the formulas for 3 of them (I won't test order unimportant, repetition allowed).