

Abstract Algebra B

Math 521B

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Review for third exam

- Be able to define precisely the following terms. Be careful about the logic in the definition!
 - transcendental element, algebraic element, minimal polynomial (Thm. 10.6).
 - algebraic extension, simple extension, finitely generated extension, splitting field.
- Know some standard examples over \mathbb{Q} and \mathbb{R} .
 - Find the minimal polynomial of some simple examples (e.g. $\sqrt{2+i}$).
See 10.2 # 1-7, 11,17.
 - Find bases for extensions: (e.g. $\mathbb{Q}(\sqrt{2}, \sqrt{3})$ over \mathbb{Q}).
 - Know the theorems and exercises about the dimension and bases of a composite extension.
(Thm. 10.4, Thm 10.10, Ex. 10.3 #1-7, 8, 9, 11, 13).
- Know how to work with finite fields.
 - Know the Freshman's rule (see also Ex. 10.6# 10, 12).
 - Know the fundamental theorem for finite fields (as stated in class, but also spread out in Sec. 10.6)
 - * There exists a field of order p^n for each prime p and $n \in \mathbb{N}$.
 - * The field is unique up to isomorphism, since it is the splitting field of $x^{p^n} - x$.
 - * The multiplicative group of a finite field is cyclic.
 - * Any finite field is a simple extension of a prime field.
 - Given an irreducible polynomial whose root α generates the multiplicative group of \mathbb{F}_{p^n} , construct the dictionary between powers of α and polynomials in α .
 - Use the dictionary to compute.