

NAME:

**Abstract Algebra B**  
**Math 521B**  
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Review for second exam

- Normal subgroups and quotient groups
  - Know the first and third isomorphism theorems.
  - Know how to compute in quotient groups.
- Direct products of groups
  - Know how to compute in direct products (componentwise!)
  - Subgroups and direct products.
    - \* If  $M$  is a subgroup of  $G$  and  $N$  is a subgroup of  $H$  then  $M \times N$  is a subgroup of  $G \times H$ .
    - \* When  $M$  and  $N$  are normal,  $M \times N$  is normal.
    - \* In this case,  $G \times H / M \times N \cong (G/M) \times (H/N)$ .
  - Not all subgroups of  $G \times H$  are of the form  $M \times N$ . Give an example.
- Classification of finite abelian groups.
  - Be able to state and use the two main theorems:  
The Fundamental Theorem of Finite Abelian Groups,  
The Invariant Factor Theorem.
  - Be able to compute the elementary divisors and invariant factors of some given group:  
e.g.  $\mathbb{Z}_{60} \times \mathbb{Z}_{75} \times \mathbb{Z}_{100}$ .
  - Suppose I give you  $n$ . How many abelian groups are there with  $n$  elements? How many have 2 invariant factors? What is the largest number of invariant factors possible for this  $n$ .
- The symmetric group.
  - Transform between cycle notation and function notation.
  - Be able to compute with each and compute the order of an element.
  - Be able to state and use the conjugacy theorem.
  - Generators of  $S_n$ : Cor. 7.48, exercises 7.9 #33, 34.
  - Know  $S_n$  and  $A_n$  for  $n \leq 4$  very well.
- Sylow theorems
  - Know the definition of  $p$ -group and Sylow  $p$ -subgroup.
  - Be able to apply the Sylow theorems to some straightforward examples: §8.3 #1-7.