

**Abstract Algebra**  
**Math 521A**  
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Review for third exam

Rings and Ideals

- Know the definitions:
  - ring, commutative, identity, field;
  - unit, zero divisor, characteristic;
  - homomorphism, isomorphism.
  - ideal, principal ideal, generators of an ideal .
- Know how to:
  - Prove that a subset of a ring is an ideal (or show that it isn't).
  - Prove that a function is a homomorphism, or isomorphism (or show it isn't).
  - Show that two rings can't be isomorphic, because they have some different structure.
  - Identify the units and zero divisors in a ring.
- Know how to prove fundamental results about ideals in a commutative ring with identity.
  - The sum of ideals is an ideal.
  - The intersection of ideals is an ideal.
  - The kernel of a homomorphism is an ideal.
  - If  $I$  is an ideal in  $R$  and  $J$  is an ideal in  $S$  then  $I \times J$  is an ideal in  $R \times S$ .
  - The annihilator of an ideal is an ideal.
- Know how to work with quotient rings.
  - If  $I$  is an ideal in  $R$ , the elements of  $R/I$  are written  $a + I$  where  $a \in R$ .
  - $a + I = b + I$  when  $a - b \in I$ .
  - Addition in  $R/I$  is defined by  $(a + I) + (b + I) = (a + b) + I$ .
  - Multiplication in  $R/I$  is defined by  $(a + I)(b + I) = (ab) + I$ .
- Know these special examples.
  - Know what the ideals are in  $\mathbb{Z}$ ,  $\mathbb{Z}_n$ ,  $F[x]$  and  $F[x]/p(x)$  where  $F$  is a field.
  - Know how to find the simplest expression for an ideal in these rings.
  - Know some examples of non-principal ideals in  $F[x, y]$  and  $\mathbb{Z}[x]$ .