

**Abstract Algebra**  
**Math 521A**  
Michael E. O'Sullivan

Review for second exam

Rings

- Know the definitions:
  - ring, commutative, identity, field;
  - unit, zero divisor, characteristic;
  - homomorphism, isomorphism.
- Know how to:
  - Prove that a subset of a ring is a subring, (or show that it isn't).
  - Prove that a function is a homomorphism, or isomorphism (or show that it isn't).
  - Show that two rings can't be isomorphic, because they have some different structure.
  - Identify the units and zero divisors in a ring.
- Know how to construct new rings from old and to compute in these rings.
  - The Cartesian product of rings  $R$  and  $S$  is a ring  $R \times S$ .
  - The  $2 \times 2$  matrices over a ring  $R$  form a ring, which we write  $M(R)$ .
  - We also have the polynomial ring,  $R[x]$  over a ring  $R$ .

Polynomial Rings

- Know the special properties of  $F[x]$ , and that it is similar to  $\mathbb{Z}$ .
  - Division theorem.
  - Euclidean algorithm.
  - Prime iff irreducible.
  - Unique factorization.
  - In  $F[x]$ ,  $(x - a)$  is a factor of  $f(x)$  iff  $a$  is a root of  $f(x)$ .