

**Abstract Algebra**  
**Math 521A**  
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Review for second exam

Rings

- Know the definitions:
  - ring, commutative, identity, field;
  - unit, zero divisor, characteristic;
  - homomorphism, isomorphism.
- Know how to:
  - Prove that a subset of a ring is a subring, (or show that it isn't).
  - Prove that a function is a homomorphism, or isomorphism (or show that it isn't).
  - Show that two rings can't be isomorphic, because they have some difference in structure.
  - Identify the units and zero divisors in a ring.
- Know how to construct new rings from old and to compute in these rings.
  - The Cartesian product of rings  $R$  and  $S$  is a ring  $R \times S$ .
  - The  $2 \times 2$  matrices over a ring  $R$  form a ring, which we write  $M(R)$ .
  - We also have the polynomial ring,  $R[x]$  over a ring  $R$ .

Polynomial rings over a field  $F$

- Know the special properties of  $F[x]$ , and that it is similar to  $\mathbb{Z}$ .
  - Division theorem.
  - Euclidean algorithm.
  - Prime iff irreducible.
  - Unique factorization.
- Know the relationship between roots and factors.
  - In  $F[x]$ , the remainder when  $f(x)$  is divided by  $(x - a)$  is  $f(a)$ .  
So  $x - a$  is a factor of  $f(x)$  iff  $a$  is a root of  $f(x)$ .
  - Know how to test whether a polynomial in  $\mathbb{Z}_p[x]$  of degree 2 or 3 is irreducible.