Convergence Tests for Infinite Series $\sum_1^\infty a_n$.

1. **n^{th} term test.** If $\sum_1^\infty a_n$ converges, $\lim_{n\to\infty} a_n = 0$.

   (This does not guarantee convergence, but if $\lim_{n\to\infty} a_n \neq 0$ or DNE, then the series must diverge.)

2. **Integral Test** (only for series with positive terms).

   Assume $f(x) > 0$, is decreasing, and continuous for all $x \geq 1$.

   Then $\sum_1^\infty f(n)$ converges <u>if and only if</u> $\int_1^\infty f(x)\,dx$ converges.

3. **Comparison Tests** (only for series with positive terms).

   (a) If $0 < a_n \leq b_n$, then $\sum_1^\infty b_n$ converges implies $\sum_1^\infty a_n$ converges.

   (b) If $0 < a_n \leq b_n$, then $\sum_1^\infty a_n$ diverges implies $\sum_1^\infty b_n$ diverges.

   (c) If $\lim_{n\to\infty} \frac{a_n}{b_n} = L$ and $L \neq 0$, then $\sum_1^\infty a_n$ and $\sum_1^\infty b_n$ either both converge or else they both diverge.

4. **Ratio Test** (only for series with $a_n \neq 0$).

   If $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = r$, then: (1) if $0 \leq r < 1$, $\sum_1^\infty a_n$ converges.

   (2) if $r > 1$, $\sum_1^\infty a_n$ diverges.

   (3) if $r = 1$, the test is <u>inconclusive</u> and the series could either converge or diverge.